

Kunihiko Kodaira

All I could do
was arithmetic

Translator's note

The following is a translation and L^AT_EXification of

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Jiewei Xiong

Preface

This book is an expanded version of *My Personal History* (私の履歴書), which was serialised in the *Nihon Keizai Shimbun* (日本経済新聞) in February of last year (1986). Since *My Personal History* is intended for general readers, I confined discussion of my own profession, mathematics, to the bare minimum required for the flow of the narrative. There is nothing quite as incomprehensible as talk of specialised mathematics.

When I look back and ask myself why I chose to specialise in mathematics and became a mathematician, I think that in the end it was simply because mathematics was the only thing I could do.

According to my mother, from a very early age I took an interest in numbers, and would repeatedly count beans as a form of play. When I was in primary school, I could do arithmetic, but I was hopeless at other subjects, a miserable pupil who disliked school. At middle school, too, I was no good at any subject other than mathematics: English, Japanese, and classical Chinese were all beyond me, and memorisation-based subjects such as history and geography were completely impossible. I remained, as before, a miserable pupil. I liked mathematics, and from my third year I was reading specialist mathematical books, but I studied simply because I found it interesting; it was not that I was studying with the intention of becoming a mathematician. I did not know that there existed a profession called a mathematician, in which one makes a living by writing mathematical papers. When I was in middle school, I aspired to become an engineer. In my fifth year, I sat the entrance examination for the First Higher School.

At that time, the entrance examinations for the old higher schools were formidable hurdles, equivalent to today's university entrance examinations, and the competition for the First Higher School was eight or nine to one. There were preparatory schools for candidates, but unlike today there was no regulation by deviation scores or anything of the sort, so anyone who wished to apply could sit the examination for the First Higher School. Had the entrance examinations been like those of today, I, who could do nothing but mathematics, would surely have been eliminated outright by my deviation score.

When I was a student at the First Higher School, the teachers all seemed wonderfully unhurried and to be enjoying themselves, and I thought I would like to become a secondary-school teacher. Even after entering the mathematics department at the University of Tokyo, all I did was study mathematics because I found it interesting – nothing more than that. After graduating from the mathematics department, I decided to have a go at physics as well, and enrolled in the physics department. In the autumn of the year after graduating from physics, I became an assistant professor at Tokyo University of Arts and Sciences. A few years later, I became an assistant professor in the physics department at the University of Tokyo, while continuing concurrently as an assistant professor of mathematics at the Arts and Sciences University. Even at that point, however, I still had not settled on any particular field of specialisation. I simply read books and papers in mathematics and theoretical physics as my interests led me, and whenever I thought of something interesting, I wrote a paper – that was all. Once I had become an assistant professor, I had no worries about making a living. I intended to spend my entire life in Japan, happily studying mathematics and theoretical physics, writing papers, listening to records, and playing the piano.

Those plans were thrown into disarray by the Pacific War. My house, my records, and my piano were all destroyed in air raids, and after the war I lived in a makeshift hut built amid the ruins, with scarcely enough to eat. Far from living happily, I sank into a miserable existence.

Once the war began, foreign literature, of course, stopped coming in, and it became completely impossible to know what kind of research was being carried out abroad. Even if I wrote a paper, it might already be known overseas and therefore utterly pointless. Moreover, as air raids intensified, it became impossible to publish specialist journals in Japan, and even if one wrote a paper there was no means of publishing it. In spite of all this, I continued to write papers. Even looking back

now, I do not really understand the psychology of those extreme circumstances. The military was shouting slogans of mass self-sacrifice, there was absolutely no sign that the war would end, and it was unclear whether one would even survive to see the post-war period – indeed, whether such a post-war period would come at all. Under such conditions, it is a mystery what I thought I was doing when I was writing papers.

The long paper entitled “Harmonic fields in riemannian manifolds”, which I wrote from the wartime years through to the post-war period, had no prospect of being published in Japan and remained unpublished. In 1948 (Shōwa 23), however, thanks to the kindness of Shizuo Kakutani, it was submitted to an American specialist journal via the Occupation forces. This paper caught the attention of Professor Weyl, and in 1949 I was invited to the Institute for Advanced Study in Princeton.

During the 1950s, the theories of complex manifolds and algebraic geometry developed rapidly, and Princeton was one of the main centres of that development. When I went to Princeton, I discovered that the paper on harmonic tensor fields, which I had written simply because it interested me, with no prospect of publication, happened to fit perfectly into and contribute to those developments. At that point I became aware that I had at last become a fully fledged mathematician, and I joined the front line of research in complex manifold theory and algebraic geometry. I was thirty-four years old.

I was fortunate enough to meet a fine collaborator, Spencer, and our research progressed steadily. Then, in September 1954, at the International Congress of Mathematicians held in Amsterdam in the Netherlands, I was awarded something called the Fields Medal. After that, I continued to live in the United States, and returned to Japan in August 1967 for the first time in eighteen years. What had once been something I never dreamed of, back when I intended to spend my whole life living happily in Japan, had become reality.

Sometimes I wonder whether living in America was not all a dream, but since my daughters speak perfect English and have shaky Japanese, it must surely have been reality rather than a dream. Even while living in America, however, there were times – such as when my wife and I went shopping together at a supermarket – when I would lose sight of her for a while and think, “Oh? I’m here on my own in a place like this – is this a dream?”

Had I not written the paper on harmonic tensor fields, for which there was no prospect of publication, or had Kakutani not been kind enough, despite that, to ask the Occupation forces to send the paper to America, I would not have been invited to Princeton. And even if I had gone to Princeton, had I not happened to meet Spencer, the research would surely not have progressed in the way it did. Spencer had been a professor at Stanford University since 1942, and by chance moved to Princeton University in the same year, 1949, that I myself went to Princeton. That I was able to meet Spencer was entirely a matter of chance.

Because mathematical research consists solely in thinking with one’s head, one has the feeling, while engaged in it, of acting autonomously. Yet when one looks back afterwards, one realises in the end that one was governed by fate.

I have continued my wandering journey through the world of mathematics, carried along by the current of fate. This book is a quiet record of the events that occurred along the way.

March 1987

At Naka-Ochiai
Kunihiko Kodaira

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I An interest in numbers

My father was born in 1885 in Yonezawa Village, Nagano Prefecture, and my mother in 1894 in Kamisuwa Town. Both of my parents were from Nagano Prefecture. Perhaps for that reason, I have somehow come to be regarded as a native of Nagano myself, and even in *One Hundred Years of Mathematics in Japan*, published the year before last, it is stated that I was born in Nagano. In fact, however, I was born in Tokyo in March 1915, and am a Tokyoite.

When I was very young, there were rental houses everywhere in Tokyo, and it seems that my parents moved house frequently. My earliest memory is of living at Gotenyama in Shinagawa, and being taken by my grandfather to watch the trains.

My next memory is from after we moved into a rented house near Sugamo Station in Koishikawa. I do not know the exact details, but I think I was four or five years old at the time. We lived there from the year after the Great Earthquake until I was in the fourth year of primary school. In the middle of a fairly large garden there were several big stones with shrubs growing among them. In summer, dozens of lizards would crawl around there. They were beautiful lizards, their backs shining with a bluish silvery sheen. If you stepped on one's tail, only the tail would come off and the lizard would escape. The severed tail would go on twitching for a long time. In one corner of the garden there was a large persimmon tree, and in autumn it bore many persimmons. They were quite good persimmons.

[母いちさんとともに]

At that time, it was rare for children to attend kindergarten. I did not go to kindergarten either, and at the age of six I entered the private Imperial Primary School. It was a small school, with only two classes in each year. Unusually for those days, it was co-educational, with each class made up of half boys and half girls. The headmaster was someone who had obtained a degree in education in America, and in American fashion there were sewing classes even for boys, who were made to sew things such as cleaning cloths. In other words, what would now be called home economics already existed. There was also, once a year, a funeral for dolls. Headless dolls brought in by the pupils were collected together, and a funeral was held for the poor dolls.

When I was four years old, my younger brother was born. Strangely enough, I remember the morning of that day. The night before, I had slept in the same room as my parents, but when I woke up in the morning I had been moved to another room. I got up and went into my parents' room, and when I tried to sit down on a bundle wrapped in a white cloth that was there, I was sharply scolded. That bundle was my brother, who had been born during the night.

[1950年代の著者]

I do not remember it myself, but according to my mother, from an early age I showed a special interest in numbers, and would repeatedly count beans as a form of play.

In arithmetic at primary school, at every lesson we were made to chant the multiplication tables like a Buddhist sutra, and were constantly drilled in calculation. Under the old shakkan-hō system of measures, one ri was thirty-six chō, one chō was sixty ken, and one ken was six shaku, so calculations of length were incomparably more difficult than calculations using today's metric system. Calculations with decimals and fractions were also practised repeatedly and at length. The most difficult were the so-called "cranes and tortoises" problems, while I think we did not learn very much that was geometrical.

When I was in primary school, I could do arithmetic, but I was no good at other subjects. On top of that, I had a quiet voice and stuttered, so I could not answer the teacher's questions properly. I was a miserable pupil, and I disliked school. Physical education was particularly dreadful. I was short, and my legs were especially short, so in races I would end up almost a full lap behind, to the point where it looked as though I were leading the race, which made me feel like crying. I also disliked composition, as I could never find anything to write about.

The year after the Great Earthquake, when I was in the fourth year of primary school, we moved

to what is now our address in Naka-Ochiai. This time it was not a rented house: my father bought a plot of land in a residential development known as Bunka-mura and built a newly constructed two-storey house.

At that time, Ochiai was a suburb of Tokyo, with many vacant lots, and on both sides of the Seibu Shinjuku Line, from Tetsugakudō to near Nakai Station, there were wide open fields.

[小学校6年生のとき、上より2列目左から3番目が著者]

When I was in the fifth year, my younger brother entered the nearby Ochiai Daiichi Primary School. Around that time, we were given a puppy by a relative and decided to keep it. It was a brown mongrel, somewhat like a setter, and very good at catching rats. We named it “Cero”. In those days, dogs were allowed to roam freely, wandering around the vacant lots in the neighbourhood, but when my brother called out loudly, “Cero, Cero”, it would come running back, wagging its tail. One day, when my brother went off to school, Cero followed him. Wondering what had happened at school, we later learned that it had gone into the classroom and sat down next to my brother during the lesson.

Cero was a female dog, and twice a year, in spring and autumn, she would give birth to five or six puppies. If all six puppies were hidden away, she would run around everywhere, whining and crying as she searched for them. Strangely enough, however, if five were hidden and just one left behind, she did not seem to notice anything at all. From this, it became clear that dogs have no concept of number whatsoever.

II My father, Gon'ichi

My father was a civil servant in the Ministry of Agriculture and Commerce, and was extremely busy. He usually returned home close to midnight, and it was not uncommon for weeks to pass without my seeing his face at all. When I was still very young, he once took me to the zoo on a Sunday, and I have the curious memory of thinking that he was some strange man I did not really know.

When an Economic Rehabilitation Bureau was established within the Ministry of Agriculture and Forestry as a measure to deal with the agricultural depression, my father became its first director (by then the Ministry of Agriculture and Commerce had already been split into the Ministry of Agriculture and Forestry and the Ministry of Commerce and Industry). The other day, at a gathering held in memory of Professor Gon'ichi Kodaira, I spoke with Mr Takeshige Ishiguro, who later served as the second director of the Rehabilitation Bureau. According to him, from morning until five o'clock in the evening my father was occupied dealing with delegations bringing petitions; only after five did he begin his regular office work, which often did not finish until close to eleven at night.

My father went from Suwa Middle School to the First Higher School, and then entered the College of Agriculture of Tokyo Imperial University (now the Faculty of Agriculture of the University of Tokyo). After graduating, he entered the College of Law of the same university (now the Faculty of Law), and completed his studies there in 1914, at the age of thirty.

[父の小平権一]

Under the education system of the time, primary school consisted of a four-year lower course, followed by a four-year upper course. After completing the four years of upper primary school, my father entered Suwa Middle School at the age of fourteen. Since the upper course at the primary school in Yonezawa Village, where my father lived, only went as far as the second year, he attended Takashima Primary School in Kamisuwa Town for the third and fourth years. From Yonezawa Village to Kamisuwa Town was eight kilometres one way, crossing a mountain pass. My father walked sixteen kilometres every day. He used to say that if he was caught in a sudden evening shower on the pass on his way home, with lightning striking close by, it was terribly frightening.

After graduating from the College of Law, my father immediately entered the Ministry of Agriculture and Commerce. After the ministry was divided into the Ministry of Agriculture and Forestry and the Ministry of Commerce and Industry, he served successively as Head of the Agricultural Policy Division, Director of the Sericulture Bureau, and Director-General of the Agricultural Policy Bureau. In 1932 he became Director of the Economic Rehabilitation Bureau, and in 1938 Vice-Minister of Agriculture and Forestry. The following year, 1939, he resigned as vice-minister and went to Manchuria (present-day north-eastern China). In 1941 he became a councillor of Manchukuo, but the following year he resigned from that post, returned to Japan, stood for election from the Third District of Nagano Prefecture, and was elected to the House of Representatives. He then became Director-General of the General Affairs Bureau of the Imperial Rule Assistance Association.

The Imperial Rule Assistance Association was an organisation intended to reflect the state of public opinion in government policy, and to ensure that the people were fully informed of the government's course of action. In reality, however, it was unable to convey to the public – indeed, even to much of the government leadership – the true situation of the Midway air and naval battle, or of the subsequent course of the war. All it could do was mobilise the people's energies under the slogan of “bringing the holy war to completion”. Disillusioned by this, my father resigned as Director-General the following year, in 1943. After the war, in 1946, he suffered the misfortune of being designated as subject to the purge from public office because of his association with the Imperial Rule Assistance Association.

The other day, biographies of my father were published. They were *The Rural, Mountain, and Fishing Village Economic Rehabilitation Movement and Kodaira Gon'ichi* (農山漁村経済

更生運動と小平権一, edited and written by Masahiro Kusumoto, Fuji Publishing, 1983) and *Kodaira Gon'ichi and Modern Agricultural Policy* (小平権一と近代農政, Nihon Hyōronsha, 1985). In fact, until I read these two biographies, I knew almost nothing about my father's career or achievements. The brief outline of my father's life given above is also based on these biographies.

Looking at the list of my father's publications included in *The Rural, Mountain, and Fishing Village Economic Rehabilitation Movement and Kodaira Gon'ichi*, one finds that he authored some forty books and around three hundred and fifty papers and other articles. I was deeply impressed by this. Someone like me does not come close to my father. I knew that he had written the substantial *Treatise on Agricultural Finance*, running to many hundreds of pages, and had obtained a doctorate in agriculture on the strength of it, but I had not realised that he had produced such an enormous body of work besides that. Of these, twenty-six books and some two hundred and eighty papers and other pieces were written in what little spare time he had amid his heavy official duties before he was designated for the post-war purge. The total number of pages of those twenty-six books alone comes to 4800. My father possessed a physical stamina that seemed never to know fatigue, but even so it is hard to imagine how, with the pressures of a life in which he returned home at midnight every night, he was able to produce such a vast quantity of writing.

My father's stamina was probably the result of the training he received as a child, walking sixteen kilometres every day for two years to and from Takashima Primary School. When I was a university student, I once said in front of him that I was tired and was scolded for it. His reasoning was that one might feel tired after walking a ri carrying a sack of rice on one's back, but otherwise there was no reason to be tired at all.

III My grandfather, Kumiji Kanai

At the end of every July, my mother would take my brother and me to her parental home in Kamisuwa Town, where we would spend the summer holidays, partly as a form of summer retreat. My mother's parents and her elder sister – that is, my grandparents and aunt – lived there. At that time, it took eight hours by train from Shinjuku to Kamisuwa.

My grandfather, Kumiji Kanai, who was born in 1857, was a member of the first graduating class of Nagano Normal School. He became headmaster of Kamisuwa Primary School at a young age, and nurtured many able people. Later he served for many years as mayor of Kamisuwa Town and as a member of the Nagano Prefectural Assembly. He was a prominent and influential debater in prefectural politics, and his sharp, well-reasoned speeches in the assembly often made the prefectural authorities tremble. At the time, his reputation within Suwa District was also very high, to the extent that a popular ditty ran, “We cannot hope to be like Lord Kanai, but at least let us become village heads” (the above according to *Kodaira Gon'ichi and Modern Agricultural Policy*).

According to records at hand, my grandfather became a primary school teacher at the age of nineteen in 1876, and at twenty-seven resigned as a headmaster to become a member of the Nagano Prefectural Assembly. That would mean that he became headmaster when he was about twenty-three or twenty-four.

[祖父の金井汲治]

The Kanai family were said to have been of the samurai class of the Suwa domain, so the house in which my grandfather lived was probably a former samurai residence. In the front garden there was a large pond, and beyond the back garden stood an earthen storehouse. The entrance hall was a spacious doma, with a reception room to the right, and beyond that, in modern terms, what would be called a dining kitchen. In the middle was a large irori hearth, from which a kettle was hung on an adjustable hook suspended from the rafters, with firewood burned beneath for cooking. This room had no ceiling, and the thick beams and the attic above were blackened with soot from the smoke of the firewood. There were several other rooms as well, and in the front garden there was a newly built detached house. In addition, there was a separate bathhouse, and since hot spring water welled up there, one could bathe at any time.

My grandfather lived a rigorously regular life. Every morning he rose at six o'clock, took a bath, and then did physical exercises there for an hour. After supper he would go out for a walk, covering four kilometres from one end of Kamisuwa Town to the other. On rainy days, instead of going into town, he would walk back and forth repeatedly along the engawa veranda of the house until he had covered four kilometres.

Well versed in Chinese studies and deeply knowledgeable about Chinese history, my grandfather would sit me on his knee and tell me all sorts of stories about China's past. I listened to them as though they were fairy tales.

My grandfather was eleven years old in the first year of Meiji, and as a child he probably attended a terakoya or similar school, where he learned classical Chinese by sodoku of hakubun. Hakubun refers to classical Chinese texts without reading marks, and sodoku means reading them aloud without any explanation of their meaning. There is a saying, “Read a book a hundred times and its meaning will become clear of itself”, and it seems that through repeated sodoku the meaning naturally came to be understood.

Later, when I was in the third year of middle school, during the summer holidays I was having trouble because I could not understand the classical Chinese texts. My grandfather said that he would teach me. Thinking this was a blessing, I took my textbook to him, but after glancing at it he merely said, “Oh – so you can't read something like this?” and in the end did not explain the meaning of a single sentence. Having learned classical Chinese through sodoku of hakubun, it probably never occurred to him that “teaching” might mean explaining the meaning of the text.

My grandfather had interests in many fields besides Chinese learning. Among the materials I

have at hand is a fragment of a notebook in which he copied out an English book on volcanology. Writing out English sentences by hand with a brush, and even reproducing detailed illustrations such as coral reefs with a volcano at their centre, must have required tremendous effort. He seems to have been particularly interested in zoology, and there were many specimens of insects and birds that he had collected while walking through the mountains and fields, all stored in wooden boxes of various sizes. On one occasion, I watched as he prepared a stuffed specimen of an animal shaped rather like a mouse. As he peeled off the skin step by step and finally pulled out the bones of the tail, the skin of the tail came off, turning inside out like a sock, which impressed me greatly.

My grandmother was a quiet, small woman. Even when something displeased her, she would merely mutter under her breath, and I never once heard her raise her voice. Every evening after nine o'clock, she would bring the notebook in which she had written that day's shopping details, together with the remaining cash, and sit formally before my grandfather to undergo an accounting inspection. If the figures were off by even a single sen, my grandfather would scold her severely, but she would do nothing more than bow her head and say, "Yes, yes." She truly lived a life of patience, obedience, and virtue.

IV The Fifth Middle School years

After graduating from primary school, I entered the Fifth Middle School (Tokyo Prefectural Fifth Middle School, the predecessor of what is now Koishikawa High School). There was an entrance examination, but since there were no such things as cram schools for primary school children, it was enough simply to revise what we had learned at school.

The Fifth Middle School was founded on the educational philosophy of its headmaster, Mr Chōshichi Itō, who believed that “education is the effort to enable a human child truly to become a human being; it is the endeavour to allow the innate gifts with which a young person is born to grow freely and to their fullest extent, and to provide every favourable condition for the development and unfolding of mind and body”. It was a school with an extraordinarily free atmosphere.

At that time, the standard uniform for middle-school students was a stand-up collar jacket, but at the Fifth Middle School the uniform was a Western-style suit with a necktie. There were also two female teachers: one taught English, and the other classical Chinese. I have heard that at a parents’ meeting Headmaster Itō remarked, “How cold it is with only men”, but among the middle schools of the time (middle schools were for boys, while schools for girls were called girls’ schools), the Fifth Middle School was probably the only one to have female teachers.

Every morning, all the pupils were gathered in the schoolyard for an assembly, at which the headmaster gave a talk. His talks ranged from everyday matters to politics and economics, but I particularly remember how he often urged us not to shut ourselves up in narrow Japan, but to go and pioneer new lands in Brazil, preaching the spirit of pioneering. “Pioneering and creation” were the motto of the Fifth Middle School.

It was also revolutionary that there was no Imperial Portrait (a photograph of His Majesty the Emperor) at the school. On ceremonial days such as National Foundation Day, instead of paying respects to the Imperial Portrait, Headmaster Itō would gather all the pupils in front of the Nijūbashi Bridge at six o’clock in the morning and lead us in three cheers of *banzai*.

From my first year through to my third, my homeroom teacher was Ms Michiko Monoi, who taught English. I remember that at the end of each term she would ask each pupil in the class how many days they had been absent that term, and write the answer into the attendance register. Daily attendance was probably not taken. The school respected pupils’ autonomy and was consistent in maintaining a free atmosphere, but regrettably Headmaster Itō passed away when I was in my fourth year.

[五中時代 (物井道子先生とその向かって左が著者)]

In the middle schools of that time, mathematics consisted of arithmetic in the first year, algebra and plane geometry from the second through the fourth years, and solid geometry in the fifth year. The fifth year of middle school corresponds in age to the present second year of high school, but there was neither calculus nor probability and statistics.

The plane geometry taught in middle school at that time was traditional Euclidean plane geometry, and through plane geometry we middle-school pupils learned logic. Euclidean plane geometry is probably the ideal teaching material for learning logic. In recent years, Euclidean plane geometry has been almost entirely expelled from elementary and secondary mathematics education, and as a result the opportunity to teach logic has been lost, which is regrettable.

Both algebra and geometry were taught using a single textbook each, used continuously from the second through the fourth years. When I was in the third year at the Fifth Middle School, my classmate Shin’ichi Nishitani and I decided that we would study the algebra and geometry textbooks thoroughly and solve every exercise from beginning to end. Once we started, it took less than half a year to reach the end of the fourth-year material.

So I went out and bought *Algebra* by Matsusaburō Fujiwara and began to read it. It was a specialist work on algebra, with the first volume running to six hundred pages and the second to eight hundred. I no longer remember clearly how far I read, but I do remember very well studying

the proof of the law of quadratic reciprocity with great determination. I also recall reading about determinants and continued fractions. I then read about Galois theory, but in the end I did not understand it. Galois theory appears at the beginning of the second volume. There is no way a middle-school student could read all the way through the second volume, so it may have been after I entered higher school that I read the part on Galois theory.

There was no particular reason why I chose *Algebra*. I must simply have happened to notice it when I went to Sanseidō in Kanda or somewhere of the sort. In the Fifth Middle School library there was *Advanced Differential Calculus* by Tanzō Takeuchi, but I avoided it, thinking it must be a highly advanced and difficult book. Had I known that it was “differential calculus for higher schools”, I think I would have read that before *Algebra*.

After that, I tried to read Hilbert’s *Grundlagen der Geometrie* in the translation by Tsurukazu Hayashi, but I could not make any headway. However, it said there that certain theorems concerning equal angles and equal lengths, which can be proved using proportions, can in fact be proved without using proportions. Once, in a geometry lesson at school, I demonstrated how to solve a proof problem – one that could easily be done using proportions – without using proportions at all, by drawing numerous auxiliary lines. All the teacher said was, “You’re a fellow who likes auxiliary lines,” and he did not praise me in the slightest.

V Algebra

It was around the time when I began reading *Algebra* that I clearly became conscious of mathematics as a subject I liked. Apart from mathematics, I was reasonably good at physics and chemistry, but in every other subject – English, classical Chinese, Japanese, geography, history, and so on – I was hopeless. As ever, my voice was quiet and I stuttered, so being called on by the teacher to read from the textbook in Japanese or classical Chinese classes was unbearable. Mr Oka, my classical Chinese teacher, scolded me severely for having such a small voice.

I was a miserable pupil even in middle school. In the classroom, I would try to make myself as small as possible so as not to catch the teacher's eye. The most depressing times were physical education and military drill. The drill instructor was a professional officer assigned to the school, and he was extremely strict.

Reading *Algebra* was by no means easy. I struggled mightily, repeating proofs I did not understand over and over again until I did, copying them out into my notebook. From that experience, I learned that even a proof one does not understand will begin to make sense naturally if one repeatedly copies it into a notebook and memorises it. In present-day elementary and secondary mathematics education, it is considered essential to make students understand first, and having them memorise proofs they do not understand is regarded as out of the question. I wonder, however, whether that view is really correct.

Thanks to the struggles I went through with *Algebra* when I was in middle school, I never had any difficulty with mathematics later, either in higher school or at university. Even so, I myself do not really know why I made such an effort to read *Algebra*, but since I was tormented by a sense of inferiority because I was useless at everything except mathematics, it may be that, unconsciously, I wanted at least to become better at mathematics than other people.

When I was in middle school, I aspired to become an engineer. I did not know that there existed a profession called a mathematician, in which one makes a living by writing papers. So it was not with the intention of becoming a mathematician that I read *Algebra*.

From 1921 to 1922, my father travelled in Europe on official business. At the time, Germany was suffering from severe inflation and the mark was cheap against the yen, so my father bought various souvenirs, including a piano. Among them was a German-made construction toy. When one opened the large box, it contained metal plates of various shapes with evenly spaced holes drilled in them, along with bolts, nuts, axles, wheels, and so on – a toy with which one could assemble cranes, trams, and the like.

I played every day, assembling all sorts of things. I learned many things from this toy. The very first thing I learned was that a triangle with sides in the ratio 3:4:5 is a right-angled triangle. When I was in middle school, I attached a small motor to a tram, wound copper wire around an iron core to make a transformer, stepped down the 100 volts of the household lighting supply to 12 volts, and ran the tram. If even toys were so interesting, I thought, how much more interesting it would be to become an engineer and assemble real things.

In later years, when I became a mathematician and constructed mathematical theories, I felt that there were similarities between building a theory and building toys. In both cases, the materials given are fixed, and unless one devises things skilfully, one cannot produce what one desires.

At the Fifth Middle School, my closest friend was Genta Tsumori. Mr Tsumori's father was an executive at Toshiba and later became its president. At that time, Kamakura was a resort area, and the Tsumori family had a villa deep in the Gokurakuji district. There were an old man and an old woman employed as caretakers, so one could stay there whenever one went. Moreover, they kept horses for Genta and his sister, with a groom to look after them. In the reception room there hung a calligraphy reading, "The best guest is one who does not say the same thing twice." At Gokurakuji there were still only two villas, and the surroundings were thick forest, filled in summer with the deafening sound of cicadas. At Shichirigahama there was only a single tuberculosis sanatorium, and

not a single other house. Inamuragasaki, where Nitta Yoshisada is said to have cast his sword into the sea and prayed for victory, still retained the atmosphere of history.

[中学校時代の級友と]

I often stayed at the villa, and Genta and I would roam far and wide around the Kamakura area. When we set out on longer excursions with packed lunches, our route might take us from Zaimokuza along the rocky shore to Kotsubo, then from Kotsubo over mountain paths all the way to Yokosuka, returning to Kamakura by train; or we might walk as far as Ōfuna and then take the train back to Kamakura. At that time, Kotsubo was still a traditional fishing village. Later, Genta Tsumori graduated from Keio University, volunteered as an officer cadet, became an officer, and was killed in action in the Sino-Japanese War. It was a great loss.

At that time, middle school lasted for five years, but it was possible to sit the entrance examination for higher school from the fourth year. Chemistry involved many calculation problems, and since I was able to do those calculations, I seem to have earned the confidence of my chemistry teacher, Mr Sekino. In my fourth year, he strongly urged me to apply to the First Higher School (the predecessor of what is now the College of Arts and Sciences at the University of Tokyo), but lazy as I was, I did not do so. The teacher said that remaining in middle school for another year would be a waste of time, but in my laziness I felt there was no need to hurry.

VI First Higher School, Science Course B

At that time, the entrance examinations for higher schools were formidable hurdles, equivalent to today's university entrance examinations, and the competition for the First Higher School was eight or nine to one. Unlike today, there was no regulation by deviation scores or anything of the sort, so anyone who wished could freely sit the examination for the First Higher School. Nowadays, the competition rate for entrance to the University of Tokyo is a little over two to one, so if anyone could simply try their luck, they might well get in by chance, but because of control by deviation scores, such freedom apparently does not exist. It is strange that the present, which ought to be freer than the pre-war period, is in fact more tightly regulated.

The main subjects in the entrance examination for the First Higher School were mathematics, English, classical Chinese, and Japanese. One remaining subject changed each year, and I think it was only announced for the first time a few months before the examination what that subject would be. To prepare for the examination, I studied at a cram school called the Nichido Lecture Course run by Kangaikata Kenkyūsha. There were frequent mock examinations. Even if one's answer was correct, marks would be deducted if the way it was written was poor, so it was not easy to obtain full marks even in mathematics mock examinations. In particular, if one devised an unusual or original proof, heavy marks were deducted.

I sat the entrance examination for the First Higher School in the fifth year of middle school, applying for Science Course B (ri-ka otsu-ru). Course B meant that German was taken as the first foreign language. Course B was further divided into two tracks: one for those intending to go on to the Faculty of Science or the Faculty of Engineering, and another for those aiming for the Faculty of Medicine. I took the examination, but apart from mathematics I did badly. As I had been taught at the cram school, I wrote my answers in large, neat characters, but in English, Japanese, and classical Chinese I could not understand the meaning of the passages in the questions at all. The questions were not of the present-day true-or-false type, but required one to translate long passages in their entirety. Naturally, I thought I had failed, so without waiting for the results I went off to stay and play at Tsumori's villa in Kamakura. Then a telegram arrived from my mother saying, "YOU'VE PASSED. COME HOME."

At that time, examination results were announced in order of merit. Strangely enough, although I had been completely hopeless in everything except mathematics, I came top of the Science B (science and engineering) track. Perhaps my answers, written with an air of understanding despite my lack of comprehension, happened by chance to hit the mark. That is why I think it is not a good idea to regulate things by deviation scores.

Science Course B was divided into two classes, and in my class of around forty students, about half were in the science-and-engineering track and half in the medical track. Since students at the First Higher School were already adults, I had assumed that there would be no taking of attendance, but in fact attendance was taken at every lesson. This was a disappointment. As I still stuttered, I would nervously dread the moment when my name was called, worrying about what I would do if I stuttered and could not answer.

At that time, the science course at the First Higher School had the feel of a school for foreign languages and mathematics. In the first year, we had ten hours of German a week. Our teacher was Professor Tsuyoshi Tachizawa. He finished the grammar in the first term, and then, as soon as the second term began, plunged straight into having us read a difficult novel in which a single sentence could run to twenty lines – a forceful teaching method indeed. Once he said, "You there – recite a, b, c (ā, pē, tsē) ... backwards from z," and I was left completely at a loss. The end-of-year examination consisted of memorising that difficult novel and writing it out in full from the beginning. During the examination, as we wrote furiously, the professor sat there calmly reading a German newspaper.

In mathematics, in the first year I studied trigonometry under Mr Hideo Watanabe. I still remember how he would lecture while gazing into the air. It is remarkable that there was enough

material to lecture for an entire year on trigonometric functions alone. Differential calculus in the second year and integral calculus in the third were taught by Mr Aramata, who was also our home-room teacher. The textbooks were *Advanced Differential Calculus* and *Advanced Integral Calculus* by Tanzō Takeuchi, which I had avoided in middle school, and they were truly outstanding and very easy to understand. After that, I studied analytic geometry under Mr Masao Tanaka, and algebra under Mr Ryūzō Kurokawa.

Of course, besides German and mathematics there were various other subjects, but in those other subjects the teachers themselves seemed unhurried. The most relaxed of all was Mr Ishikawa, who taught botany. Even when taking attendance, if someone said, “That student has gone to rowing practice,” he would reply, “In that case, I’ll mark him present.” On just one occasion, however, he did admonish a student who was lying sprawled on his desk reading a newspaper during the lecture, saying, “Come now, that really is too much.”

After entering the First Higher School, I read the *Iwanami Lecture Series in Mathematics* and Professor Teiji Takagi’s *Lectures on Elementary Number Theory*. I also continued reading *Algebra*, which I had begun in middle school.

From time to time I visited Mr Aramata at his home, where we would drink beer and he would treat me to dinner. His wife was a graduate of the Ueno Music School, and I heard that she had specialised in vocal music.

When I was in middle school I had aspired to become an engineer, but when I was a student at the First Higher School I wanted to become a higher-school teacher. That was because Mr Aramata seemed to be enjoying himself so very much.

VII Entering the University of Tokyo

In 1935, I graduated from the First Higher School and entered the mathematics department of the University of Tokyo. At that time there were no such things as cram schools for university entrance examinations, and if one studied properly at higher school, entering university was not particularly difficult.

The mathematics department admitted fifteen students. There were only five chairs, with Professors Teiji Takagi, Senkichi Nakagawa, Sōichi Kakeya, Tanzō Takeuchi, and Joichi Suetsuna; the associate professors were Masatsugu Tsuji and Shokichi Iyanaga; there was a single assistant, Mr Shunji Kametani; and besides them there was one elderly man and one elderly woman who worked as caretakers. There were no administrative staff at all, and Mr Kametani also doubled as the librarian. Given that there were no administrative staff whatsoever, there must have been very little in the way of administrative work in the mathematics department. All the professors appeared unhurried and at ease.

[東大入学時の著者]

The lectures for us first-year students included calculus by Professor Takagi, algebra by Professor Suetsuna, and geometry by Professor Nakagawa. The calculus lectures were scheduled four times a week, from eleven to twelve on the timetable, but Professor Takagi would arrive at the mathematics building around ten past eleven, then sit calmly drinking tea in the caretakers' room. The lecture would begin at eleven thirty and end exactly at twelve, as scheduled. I still remember the impression made by Professor Takagi's enormous ears as he wrote formulas on the blackboard. The content of the lectures was almost the same as that of the *Introduction to Analysis* he wrote for the Iwanami Lecture Series in Mathematics. It is astonishing that with four lectures a week, each only thirty minutes long – two hours a week in total – he covered the whole of *Introduction to Analysis* (excluding the Lebesgue integral in the present edition) in a single year.

The calculus exercises were taught by Professor Iyanaga. In the Iwanami Lecture Series there was *Foundations of Geometry* by Shokichi Iyanaga. Before entering university, I had imagined that the author of such a difficult book, with its forbidding title, must be a stern, broad-shouldered, formidable-looking professor. In reality, however, he was unexpectedly a tall, gentle man, resembling Chopin as portrayed in the then-famous film *Song of Farewell*.

At the very first algebra lecture, Professor Suetsuna drew on the blackboard a large circle with what looked like a little moustache. This, astonishingly enough, was the German letter K , representing a field (that is, a system in which addition, subtraction, multiplication, and division are defined). The algebra exercises were taught by Professor Suetsuna himself. These exercises were frightening. If, when called upon, one said straight away, "I can't do it," he would merely give a wry smile and that would be the end of it. But if one went up to the blackboard, began to solve the problem, and then got stuck halfway through, he would scold severely, saying, "What are you dithering about?"

In addition, mechanics exercises were compulsory. Since there were exercises, there must have been lectures in mechanics as well, but I have no memory of the lectures themselves. The mechanics exercises were once a week, from one o'clock until five or six, and were a severe ordeal, like an ascetic trial.

At that time, Japanese mathematics was at a turning point, shifting from classical mathematics to modern mathematics, and subjects that would be unthinkable today were compulsory. One of these was Professor Nakagawa's geometry. He lent each student a copy of Salmon's *Analytic Geometry*, a decidedly old-fashioned textbook. It was a fairly thick book – around three hundred pages, I think. Its content was the theory of quadric surfaces in three-dimensional space. It is curious that as many as three hundred pages could be written on quadric surfaces alone, and in addition the book contained a large number of exercises. We were made to work through these in the geometry exercises, and they were extremely difficult. This kind of geometry has now been almost entirely forgotten, but the mathematics that we are currently researching will probably also be largely for-

gotten a hundred years from now. When one thinks of it that way, it produces a rather strange feeling.

[東京大学数学科, 下段中央は高木貞治先生、向かって右端は弥永昌吉先生、続いて末綱愨一先生.]

In the mathematics departments of the old imperial universities, all subjects except mechanics were mathematics, and since mechanics too was an application of mathematics, it was enough to study mathematics alone. And, as one would expect at university, students were treated as adults: attendance was not taken, and even during examinations the professor would merely come in two or three times to have a look, with no invigilators present. After becoming a university student, there were no longer any subjects that made me feel miserable, and for the first time I was happy to feel that I had shed my sense of inferiority and become a full-fledged person.

As there were only fifteen students in each year, a very small number, we first-years soon became close to one another. On the day of Professor Suetsuna's algebra examination, the February 26 Incident broke out and the examination was cancelled, so we were delighted and went off to Ueno Zoo.

When we became second-year students, Makoto Abe entered the mathematics department. Abe had taken the entrance examination for the physics department a year earlier but had been failed for health reasons, although his examination score was said to have averaged ninety-six points – a superhuman achievement. He was extraordinarily learned, and there seemed to be nothing he did not know. Later, after I myself had become an assistant professor in the physics department, I had occasion to look at past entrance examination scores, and in an average year the highest score was around seventy points; ninety-six points was unprecedented. We became as close to Abe as though he were a classmate from our own year.

VIII A habit of skipping lectures

In my first year I attended lectures conscientiously, but after a while in my second year I began to miss them more and more. It struck me that listening to lectures for two hours a week was somehow inefficient. Excluding the summer and winter vacations, lectures ran for eight months of the year, and at two hours a week that amounted to only sixty-four hours a year. If one studied for eight hours a day, that would be finished in eight days.

So I decided that as far as university lectures were concerned, I would borrow my friends' notebooks and study from them before the end-of-year examinations. Instead, I went to Maruzen and bought various books, reading them one after another. The very first book I read was on the Lebesgue integral. I have forgotten both the title and the author, but it was a small book with a red cover. From around this time, I acquired the habit, when reading mathematics books, of thinking up alternative proofs, or constructing examples and counterexamples as I went along. I remember finding it interesting to construct examples of Lebesgue non-measurable sets on the real line with special properties, though I can no longer recall what those examples were.

[東大数学科時代の学修簿. 成績とともに担当教員の署名, 捺印がある.]

Although I skipped lectures, I think I still went to the university every day – or perhaps it was every other day. In any case, I would go with the others to the Second Dining Hall, eat ice cream, and chat. The regulars at these conversations were my classmates Kiyoshi Itō, Takayoshi Kawada, Shigeru Furuya, Hideo Nakamura, Issei Shiraiishi, and also Makoto Abe, who was a year below us. The set lunch at midday cost fifteen sen, the special set twenty-five sen, and rice was free, placed on the table in a wooden tub. If there was even the slightest speck of dirt on the ice cream, there were bold characters who would summon the waitress, have her bring a new one, and then eat both the contaminated one and the replacement.

To give a small illustration of Makoto Abe's encyclopaedic knowledge: if you went for a walk with him, he could tell you the names of every wild plant growing by the roadside; if you went to the cinema, he would explain what century the buildings appearing on screen belonged to and what architectural style they were in. He was also a fine pianist and well versed in music theory. One day, as usual, we were all licking ice cream and chatting in the Second Dining Hall when Hideo Nakamura, who prided himself on being knowledgeable, tried to recite the *Jinnō Shōtōki* from memory, but got stuck halfway through. Kiyoshi Itō continued from there, but he too stumbled. Then Abe carried on from where they had left off. From that day on, Nakamura was regarded as a self-styled man of learning, while Abe was recognised as the real thing.

Summer vacation came. I intended to spend it studying a thick German book entitled *Topology* by Aleksandrov and Hopf, but since there were no air conditioners yet, the heat was unbearable. At night it was cooler if one went out into the garden, but then one was attacked by swarms of mosquitoes. I do not remember the details clearly, but according to my mother, I apparently hung a mosquito net in the garden and studied inside it. In the end, I pleaded with my father and obtained the considerable sum of one hundred yen, went to Karuizawa, and stayed there studying at the Karuizawa Hotel, which stood in the centre of Old Karuizawa.

The reason I chose Karuizawa was that I could not forget the wonderful impression it had made on me when I had spent one summer there as a child. That summer, for some reason, my mother did not go to Kamisuwa, but instead took my brother and me to Karuizawa, where we stayed together with my father's younger brother's family in a villa we rented at the foot of Mount Hanareyama.

At that time, perhaps because Karuizawa had been burnt out by the great Tenmei eruption of Mount Asama, there were hardly any trees. Instead, a vast grassland spread out in every direction, filled with blooming flowers such as matsumushisō, waremokō, bellflowers, and daylilies, all competing with one another. From the second floor of the villa, one could look out over the whole of southern Karuizawa at a glance. At the foot of Mount Hanareyama there were still only a handful of villas. The weather, too, was different from today, with many clear days, and the shadows

of towering summer clouds drifting quietly across the grasslands. It was a truly magnificent view. Almost every day there were evening showers accompanied by thunder. Mist would sometimes roll in, but the air was fresh and invigorating.

The Karuizawa Hotel stood roughly in the middle of Old Karuizawa. At that time, on the Shin'etsu Line it took an hour to travel from Yokokawa to Karuizawa, hauled by an Abt-system electric locomotive. As soon as the train emerged from the Usui Pass tunnels, the cool air of the highlands would stream in through the windows. It was the refreshingly cool Karuizawa of former days.

Even then, hotels in Karuizawa were not cheap, so with one hundred yen I could not stay very long. I stayed for ten days, after which I went on to my grandfather's house in Kamisuwa via the Koumi Line, stayed in the detached building, and continued studying there. On the Koumi Line, a flea fell from the luggage rack of the carriage onto the back of my neck and crawled down my back, biting me as it went. That was truly exasperating.

After *Topology*, I read *Algebren*, an algebra book by van der Waerden. Then I wrote a short paper.

The lectures for second-year students included function theory by Professor Tanzō Takeuchi and differential equations by Professor Sōichi Takeya. When I once quietly peeped in during a differential equations exercise class, I saw that only five or six students were present, and Professor Takeya was comparing the faces of those in attendance with students' photographs. I thought this was troublesome, but for some reason I still did not attend.

When the end-of-year examinations approached, a few weeks beforehand I borrowed Takayoshi Kawada's notebooks and copied them out. Kawada's notes were extraordinarily detailed; everything the professors had said was written down. Of course, there were no photocopiers, so they had to be copied by hand. As I organised the notes and carefully transcribed them, the content of the lectures naturally entered my head through that process alone. There was someone who later borrowed the notes I had copied, but I cannot remember who it was.

IX A moratorium man

In the third year, only “Mathematical Seminar” was compulsory, and apart from that one did not have to do anything. In the “Mathematical Seminar”, each student was assigned a supervisor and was to begin mathematical research by reading books and papers. *Kōkyū* simply meant a seminar.

For my third-year seminar I was assigned to Professor Iyanaga. At that time I was studying algebra from van der Waerden’s book, so I thought I would do an algebra seminar. Together with Takayoshi Kawada, who was studying from the same book, I went to Professor Suetsuna’s house to ask him to supervise our seminar. At the time, it seemed that he agreed, but a few days later I received a letter from Professor Suetsuna saying, “For your seminar, study geometry under Professor Iyanaga.” I immediately took that letter and went to Professor Iyanaga’s house to ask him to supervise me.

Strangely enough, I have no memory of what we did in the seminar. No scene of the seminar at all comes to mind. According to Professor Iyanaga, I read *Topology* by Aleksandrov and Hopf in this seminar. That was the very book I had stubbornly struggled through the previous summer, even going as far as Karuizawa to read it. Perhaps because it was my second time reading it in the seminar, it failed to leave much of an impression.

In the summer vacation of my third year, I climbed Mount Okuhotaka together with Shigeru Furuya and one of his friends, the three of us. Furuya liked mountaineering and had already climbed many high mountains of over three thousand metres, but this was the first and last time I ever climbed such a high mountain. When we set out from Kamikōchi the weather was fine, but it gradually worsened, and by the time we reached the ridge we were completely surrounded by cloud, with visibility reduced to only a few metres. Had it been clear, the view must have been magnificent, but all I could see were rocks a few metres ahead. I had no idea at all which direction we were walking in, but Furuya apparently knew, and by evening we reached the mountain hut on Mount Okuhotaka.

On the train back on the Chūō Line, I heard people around us saying things like, “Next time the red notices will come,” and something felt odd. When I returned to Tokyo and read the newspapers, I discovered that while we were climbing the mountain the Marco Polo Bridge Incident had occurred, and the war had begun.

After graduating from the mathematics department, I took the entrance examination in the usual way and entered the physics department. The examination included chemistry, which was the subject I was worst at. Thinking there was no way I could manage it, I went to see Professor Terazawa, the head of the physics department, and said, “I can’t do chemistry.” He replied, “In the physics department entrance examination, chemistry is not given much weight, so you can get in even if you score zero in chemistry.” Reassured by this, I studied physics for about a month and then sat the examination.

The reasons why I re-entered the physics department were, first, that books such as Hermann Weyl’s *Group Theory and Quantum Mechanics* and von Neumann’s *Mathematical Foundations of Quantum Mechanics* were being published at the time, and it seemed that the relationship between mathematics and physics was becoming ever closer; and second, that, in today’s terms, I myself was a moratorium man. The Marco Polo Bridge Incident was expanding steadily, and even if one graduated it was impossible to see what lay ahead. Today’s moratorium students postpone graduation by repeating a year, but I postponed graduation by entering the physics department.

At that time, theoretical physics at the University of Tokyo had a strong mathematical-physics flavour. Several of the compulsory subjects were shared with the mathematics department, and whether it was relativity or quantum theory, what students struggled with was the mathematics used in them. For me, having graduated from the mathematics department, the physics department was therefore easy. Moreover, for some of the mathematics courses intended for physics students, I asked the instructors to exempt me from the examinations, and they kindly did so. After a lecture, I

would ask the professor in the classroom, “Please exempt me from the examination,” and he would reply on the spot, “Yes, all right.” How they were able to exempt a student from an examination on their own authority, without consulting a faculty meeting, and how they then assigned a grade to a student who had been exempted, seems mysterious when I think about it now, but at the time I thought nothing of it. It was all extraordinarily carefree.

[東大時代の著者]

In the first year, physics experiments were compulsory. I remember very clearly that the first session was practice in soldering, but I have no recollection at all of what experiments we did from the second session onwards. There was also something called exercises in mathematical physics, which, like the mechanics exercises I had had in the mathematics department, was an ordeal akin to ascetic training. In the second year, I went to hear lectures on meteorology by Professor Sakuhei Fujiwhara, who was famous at the time as the “weather doctor”. At the beginning of the first lecture, he said in a leisurely tone, “In the first term I shall talk about dull things. When the number of students decreases in the second term, I shall talk about slightly more interesting things.” I was taken aback and gave it up after just one lecture. I also attended lectures on astronomy by Professor Hagiwara, but they were delivered at a tremendous speed and I could not understand anything at all. After the lecture, when I was drinking tea in the caretakers’ room, the professor came in to have tea as well and said with great satisfaction, “Well then, you didn’t understand it, did you?” That was when I realised that lectures are not necessarily given in order to make students understand.

X A connection with the piano

In the midst of all this, I reached my third year. For my third-year seminar I studied field theory under Professor Takuzō Sakai. Field theory had only just begun at that time, and there were no books yet, so we read a number of papers.

Even after entering the physics department, I continued my research in mathematics alongside my studies in physics. During my time in the physics department I wrote eight papers. Of these, six were short papers published in the *Proceedings of the Japan Academy* in a European language. The final one, however, was a long paper on Haar measure. I first published it in Japanese in the *Zenkoku Shijō Sugaku Danwakai* (National Mathematical Discussion Forum in Print), then translated it into German over the summer and autumn of my third year, and had Professor Iyanaga revise my German before submitting it to the *Journal of the Physico-Mathematical Society of Japan*. At that time, the Mathematical Society of Japan and the Physical Society of Japan were united under the name of the Physico-Mathematical Society of Japan. Perhaps because it had ample funds, or perhaps because there were few submissions, my long paper was printed within a matter of months.

To return briefly to an earlier point, I began learning the piano when I was in the third year of middle school. The piano my father had bought in Germany in 1922 had been left standing in our drawing room. When I tried playing it a little around my third year of middle school, I found that I could read music surprisingly well, and soon became able to play simple sonatinas. At that time, the family of Mr Renji Ishikawa, a professor of German literature at Tokyo Higher School, lived next door. He was the younger brother of my mother's elder sister-in-law, and I called him Uncle Ishikawa. He was fond of music, and said that if I was going to play the piano, it would be better to study with a professional. He then introduced me to one of his former students, Tokuji Nakajima, a University of Tokyo student who played the piano. Once I began lessons, I found that every day I had to start with scales and arpeggios and practise for two hours. It was quite an ordeal. I sometimes regretted having started at all, but I persevered and went once a week to Nakajima's house for lessons.

After a while, Nakajima graduated from the University of Tokyo and took up a post in the provinces, so I began studying instead with his elder sister, the violinist Tazuko Nakajima. I do not clearly remember how long I studied with her, or exactly which pieces I learned, but I vividly recall studying Mendelssohn's *Rondo Capriccioso*. Around that time, the pianist Benno Moiseiwitsch came to Japan, and I went with Tazuko-sensei to hear his piano recital. *Rondo Capriccioso* was included in the programme. The tempo was astonishingly fast, and I was left speechless. This was the first piano recital I had ever heard in my life. Afterwards, Tazuko-sensei told me that it would really be better to study the piano with a pianist, and through her introduction I went to study with the wife of Tetsutarō Kawakami. It is only to be expected that one is scolded in piano lessons, but there was something particularly dispiriting about Mrs Kawakami's way of scolding. After a few months, I grew tired of it and gave up.

[リサイタルの後で。前列右端が著者、同右から5番目が外山滋さん。]

Some time later, again through Tazuko-sensei's introduction, I began lessons with Akiko Sawazaki. This time I continued for a comparatively long period, even after Akiko-sensei married Motonari Iguchi. However, in my first year in the mathematics department I was hospitalised for an appendectomy and took a break from lessons, and that was the end of it. I remember studying Beethoven's Piano Sonata No. 17 and Schumann's *Humoreske* with Akiko-sensei. The last piece I was working on was Chopin's Ballade No. 1.

I gave up taking piano lessons, but before long I began to be asked each year to provide violin accompaniment at the recitals of Tazuko Nakajima's pupils. This was when I was a student in the physics department. Among her pupils were small children, and it was somewhat awkward to ask a professional pianist to accompany them. I was apparently chosen because I could be asked casually and was able to sight-read.

Among those whom I accompanied were Shigeru Toyama, Takako, the daughter of Professor Hiroshi Tamiya, Toyoko Ueno, and Seiko, the younger sister of Professor Iyanaga. I have a photograph from a recital in 1940: at that time Shigeru was five years old, and Takako was ten, with a round face and doll-like cuteness. Shigeru Toyama is now active as a violinist, and Takako later switched to the piano and is now an associate professor at the Tokyo University of the Arts. It was through this connection of accompanying violinists that I later came to marry Seiko.

XI Memorable masters

After Moiseiwitsch, I heard a piano recital by Uramaru Nobeji. I remember that the sound of the piano was extremely quiet. After that, I often went to piano recitals, but the pianists whose names I still remember are, among Japanese performers, Sonoko Inoue, Motonari Iguchi, and Chieko Hara; and among foreign pianists, Arthur Rubinstein, Ignaz Friedman, Wilhelm Kempff, and Emil von Sauer. As for orchestral concerts, I attended the regular performances of the New Symphony Orchestra (the predecessor of today's NHK Symphony Orchestra). The conductor was Hidemaro Konoye, later replaced by Joseph Rosenstock. Tazuko Nakajima was a member of the New Symphony Orchestra and played the violin there.

I remember that Rubinstein gave piano recitals for four consecutive evenings at Hibiyu Public Hall. According to his autobiography, this was in the spring of 1935. I went to hear only one of the evenings, but the audience turnout was poor, with about a third of the seats empty. I have forgotten the works on the programme, but what remains vividly in my memory is the encore, Falla's *Dance of Fire*, which he played while alternately raising his hands so high that they seemed almost to reach the ceiling. This appears to have been one of his trademark gestures, for when I later heard him again in Baltimore in the United States, he once more played the same *Dance of Fire* as an encore, raising his hands in exactly the same way.

At Wilhelm Kempff's recital, during the interval he invited themes from the audience, chose one of them, and then improvised on it. The theme selected was *Echigo-jishi*. The performance began with a quiet fugue and went on for as long as fifteen minutes, ending with a majestic set of variations that called to mind a Beethoven symphony. I was utterly impressed. How such a thing could be done was beyond my imagination. This was the first and last time in my life that I ever heard an improvisation.

Walter Gieseking, who fled the Nazis and settled permanently in Japan, was someone I heard many times and who left a deep impression on me. His performances of Liszt's *Gnomenreigen*, the Paganini Études, Chopin's Preludes and Études, the Nocturne in G minor, and Schumann's *Symphonic Études* still ring in my ears. He played *Gnomenreigen* entirely in pianissimo, from beginning to end, producing a strange effect as though the whole piece were veiled in a spring haze. In the Paganini Études, I was overwhelmed from the very opening arpeggios and chromatic scales of the first étude. There is a passage at the end of Chopin's G-minor Nocturne where the melody rises from G up to a C two and a half octaves higher; when he played it, it sounded as though it were ascending from the depths of the earth to the heights of heaven. When Kreutzer played, the low notes sounded endlessly deep and the high notes endlessly high – it was as if the range of the piano itself had doubled, which I found quite mysterious.

Foreign pianists were on an altogether different level from Japanese pianists; even the very sound of the piano was different. Looking back now, Rubinstein, Friedman, Kempff, and Kreutzer were all pianists whose names have gone down in history as masters, so it was only natural that they played superbly. But in my ignorance, I assumed that all foreign pianists played at that level. That was a serious mistake. As a result, even after moving to Princeton in 1949, I took it for granted that whenever I wished I would be able to hear such wonderful performances, and I made no effort at all to go as far as New York – only an hour away by car from Princeton – to attend concerts. I never heard Arturo Toscanini, Artur Schnabel, or Vladimir Horowitz in his prime. I think now what a terrible shame that was.

As for the regular concerts of the New Symphony Orchestra, all I really remember is being bored by hearing Mahler's *Das Lied von der Erde* over and over again; finding the performance of Ravel's *Alborada del gracioso*, conducted by Joseph Rosenstock, so brilliant that I had difficulty suppressing laughter while listening; and being deeply moved every time I heard Beethoven's Ninth Symphony.

I began listening to records when my uncle bought me an electric gramophone, around my fourth year of middle school. Both the gramophone and the records were later destroyed in air

raids, but the one record that remains particularly vivid in my memory is the *Concert Arabesque on "The Blue Danube"* by Schulz-Evler, played by Josef Lhévinne.

This piece is a concert arrangement of Strauss's *The Blue Danube* Waltz. These days, it seems that no pianists play arrangements of this kind, but at the time it was a famous virtuoso showpiece and was performed quite often. I heard performances of this piece many times at Hibiya Public Hall. Once was at a recital by Ignaz Friedman; another time was at a farewell recital given by a foreign professor at the Ueno Music School on the occasion of his return home at the end of his appointment; and the third time was when an arrangement of the piece for two pianos was played by Joseph Rosenstock and Sonoko Inoue. At that time, I had no idea what kind of pianist Josef Lhévinne was, but the record was superb and an utterly delightful performance, so I listened to it over and over again.

Not long after the end of the war, when I heard this Lhévinne record again on the radio in Yonezawa Village in Suwa, where we had been evacuated, it filled me with deep emotion: ah, at last we have returned to a peaceful world in which one can once again listen to music as before. In the radio commentary at that time, I learned that Lhévinne had died during the war.

Later, after I came to live in America, I obtained in 1955 a record released by Victor entitled *The Art of Josef Lhévinne*. Reading books about pianists, I then came to realise that Lhévinne had in fact been a truly top-rank artist. The record was a compilation of Lhévinne's SP recordings transferred to LP, and included Chopin's Études in thirds, octaves, and "Winter Wind", the *Heroic* Polonaise, Schumann's Toccata, and, of course, Schulz-Evler's *Concert Arabesque on "The Blue Danube"*.

The playing is superhuman. One can tell this by listening to it side by side with the recordings by Maurizio Pollini, which are now regarded as model performances of Chopin's Études. Take the Étude in thirds, for example: compared with Lhévinne, Pollini's performance sounds almost like a conservatoire student practising. If one listens only to Pollini, his scrupulous fidelity to the score seems impeccable, and one comes to think that this is simply what the Étude in thirds is like. But when one hears Josef Lhévinne, it is as though one's eyes are opened: so this is how the Étude in thirds ought to be played, and one awakens to its beauty. Pollini plays faithfully to the score, but his reading is still shallow; Lhévinne's reading is deep – that, I think, is the difference.

Lhévinne was a contemporary of Sergei Rachmaninov at the Moscow Conservatoire. According to *The Great Pianists* by Schonberg, Josef Hofmann and Rachmaninov were the greatest virtuosi of Lhévinne's generation.

Rachmaninov's playing can be heard on records. In the Victor release *The Art of Rachmaninov* there is Schumann's *Carnaval*. The overwhelming power of the performance – driven through to the end without a moment's respite, at a flying tempo, in the final movement, *March of the Davidsbündler against the Philistines* – is astonishing. When one listens to this side by side with it, even Arthur Rubinstein's recording sounds like a student practising. Had Rachmaninov's recordings not survived, we might never have known that Schumann's *Carnaval* is such fantastically imaginative music. One cannot grasp the very essence of music unless one hears it played by a master. It is a strange thing indeed.

XII The outbreak of war

After graduating from the physics department, I became a research associate there and began giving lectures. I have no recollection at all of what I lectured on, but I vividly remember the strange feeling I had the first time I stood at the podium and looked out at the students in black uniforms listening solemnly to my lecture. At the end of that year, the Pacific War broke out.

A year and a half later, in September 1942, upon the recommendation of Professor Masao Sugawara, I was appointed associate professor in the mathematics department of Tokyo Bunrika University. I continued, as before, to lecture in the physics department at the University of Tokyo. At Bunrika University I was responsible for lectures in mathematics and for supervising student seminars, but there were no departmental meetings or faculty meetings. It sounds odd to say that there were none, but presumably Professor Sugawara took all such administrative matters upon himself so that I could devote myself entirely to mathematics.

On 30 May 1943, with Professor Sakuhei Fujiwhara acting as go-between, I married my present wife, Seiko. For our honeymoon we went to Gōra in Hakone, taking our rationed rice with us. By that time food shortages were already severe, and unless one brought one's own rice, one could not stay at a hotel even if one paid.

In January of the following year, 1944, I became associate professor in the physics department at the University of Tokyo. I continued concurrently as associate professor in the mathematics department at Bunrika University. In March our eldest son, Kazuhiko, was born. In the physics department I was, naturally, in charge of mathematical physics.

[結婚式で(昭和18年)]

From around the autumn of that year, the air raids on Tokyo gradually intensified. Whenever the air-raid siren sounded, we would take shelter in the basement of the physics building. The silver B-29s, flying in formation at an altitude of ten thousand metres across the transparent blue sky, were extraordinarily beautiful. It was impossible to think that they were the work of the same human beings as ourselves, cowering in the dim basement. It felt as though we were being attacked by beings from outer space, and I felt no particular hostility toward the enemy.

[帝国ホテルの披露宴の献立、戦時中でもあり質素である.]

From about that autumn, it was decided that the women and children of the Iyanaga family would be evacuated to the villa in Karuizawa. Including my wife and eldest son, five women and seven children began living there communally. The rationed rice was not enough, so my wife would ride her bicycle as far as the area around Lake Shiozawa to obtain supplies. It was all barter: for example, one silk kimono could be exchanged for one shō of azuki beans; and she was told, "Next time, bring a coat." We even tried keeping a goat so that the children could drink its milk, but the poor creature soon died after overeating bran. We had not known the elementary rule of goat-keeping – that one must not allow it unlimited amounts of feed.

During the winter holidays three men joined us, making eight adults in all, as I recall. The villa had been built as a summer retreat, and it was no easy matter to endure the severe winter of Karuizawa there. The temperature fell to minus twenty degrees Celsius. We kept the water running in a thin stream so that the pipes would not freeze, yet one night a water pipe burst, and we had great difficulty stopping the gushing water. There was firewood, so we could heat the traditional Goemon bath, but thick ice formed on the bathroom walls and did not melt throughout the entire winter.

The greatest difficulty was the toilet. The flush toilet was frozen and unusable, so we all used the old-fashioned pit latrine in the maid's room. But it froze from the edges inward, and a mountain of frozen excrement gradually formed and kept growing, until at last its summit rose higher than the floor. I tried to break it with all my strength, striking it with a hammer, but the iceberg did not budge; only a small chip came off the top. A fragment flew into my face; it felt icy cold, but there was not the slightest smell. I thus discovered that when filth is cooled to minus twenty degrees Celsius,

even its stench freezes away.

In the following year, 1945, the air raids became more and more frequent. If we had to dive into the basement every time the siren sounded, we could hardly conduct any classes at all. I began to think that somehow we ought to evacuate the physics department to the countryside, and when I consulted my father, he said he would arrange a place for us. So at a dinner gathering of the physics department I proposed, "I think we should evacuate the department to the countryside." Someone asked, "Do you have a place to go?" I replied, "My father says he will arrange it." "In that case, let us evacuate," they said, and the decision was reached unanimously on the spot.

I was astonished. I had never imagined that so large a matter would be settled so easily. Perhaps Parkinson's Law was at work, and precisely because it was such a major issue it was decided all the more readily. But once it was decided, it became my responsibility to carry it out. Moreover, the mathematics department also decided to join the evacuation. The trouble was that my administrative ability was zero. There was nothing for it but to rely entirely on my father to negotiate with the village office in Suwa, the elementary school that would lend us classrooms, and the inn that would provide lodging. As for me, all I did was follow my father's instructions and pay courtesy visits to the village mayor, the principal of the elementary school, and the others.

In short, I did nothing at all; yet I seem to have given the impression that although I looked like someone incapable of doing anything, if given the chance I could manage fairly well. This, it appears, later became one of the reasons why I was chosen as Dean of the Faculty of Science.

By nature I am a lazy man. In the *Life Nature Library* I once read that the sloth of South America survives by being lazy to the utmost – hanging motionless from a tree branch until moss grows over its body and it becomes indistinguishable from the vegetation. It is said to be the sole descendant of the ancient creature Megatherium, which succeeded in surviving precisely by such thoroughgoing indolence. When I read that, I was deeply moved and thought: this is exactly my ideal! Such is the extent of my laziness. Needless to say, I detest any position with the word "chief" or "head" attached to it. That I should nevertheless have been elected dean must be a case of poetic justice.

The evacuated university was divided between two locations: Chino and Osachi Village, deep in the interior beyond Shimosuwa. The second- and third-year mathematics students were in Chino, while the first-year mathematics students and the physics students were in Osachi Village. In Chino, the students lodged on the second floors of a public bathhouse and a sushi shop. In Osachi Village, a small hot-spring inn in the mountains served as the students' dormitory. Professors who had evacuated with their families rented rooms in corners of farmhouses.

Though we had escaped the air raids by evacuating, we suffered greatly from the food shortage. We had assumed that food conditions in the countryside would be better than in Tokyo, but this proved entirely mistaken. Those who have never experienced it can never truly understand: to have nothing to eat is a profoundly miserable thing. And yet everyone studied diligently. Judging from the fact that a number of outstanding mathematicians emerged from this evacuated class, there seems to be little correlation between living conditions and scholarship.

XIII The end of the war

Our family evacuated to my father's birthplace in Yonezawa Village. It was an old house where my grandparents had once lived. Each day I walked six kilometres to Chino, took the train from Chino to Shimosuwa, and then walked another kilometre from Shimosuwa to the elementary school in Osachi Village where the physics department had borrowed classrooms. In all, I walked fourteen kilometres a day.

In the air raid of 13 April, my father's house in Naka-Ochiai, Tokyo, was burned down. My father, his secretary Mr Hideo Watanabe, and our maid Hanayo, who were in the house at the time, fled from the flames and ran for their lives. Before dawn they arrived, looking like beggars, at the home of Mr Kimio Morimoto in Shōtō, Shibuya. After spending several days there discussing what to do next, it was decided that my father, Mr Watanabe, and Hanayo would move into a dormitory called Risshisha in Chitose-Funabashi, Setagaya. My mother was in Yonezawa Village and thus escaped the air raids.

On 15 August of that year, the war came to an end. Even though the war we had been told was certain victory ended in defeat, no particular commotion occurred. In their hearts, perhaps everyone had expected defeat.

[疎開先の米沢村の家]

My younger brother had been conscripted into the army, rising from officer-cadet to commissioned officer. Fortunately, he remained in Japan, and around the time of the war's end he was stationed with a shipping unit in Ujina, about six kilometres from Hiroshima. On the morning of 6 August, while he was in the unit's office, he heard a tremendous explosion and rushed outside, where he saw the mushroom cloud clearly. He ran back to the office to fetch his camera and hurried out again, but by then the mushroom cloud had lost its shape and become cylindrical. That night he went into Hiroshima and, for ten days, helped clear away the corpses.

A shipping unit was a logistical base for army units dispatched overseas, and it was well supplied with materials such as timber, rice and sugar. After the war, the weapons and ammunition were handed over to the American forces, and the remaining supplies were distributed to soldiers returning to their home towns. I am told that one soldier repeatedly transported timber to his home near Hiroshima and built a house, saying to my brother, "If you ever come to Hiroshima, please be sure to visit." Having settled such matters, my brother returned to Yonezawa Village at the beginning of October. He brought sugar with him, but no rice. Apparently he did not fully realise how severe the food shortage was; he later said he ought to have brought at least one sack of rice.

Makoto Abe, who had enjoyed a reputation as a brilliant scholar, was the eldest son of Abe Yoshishige, then headmaster of the First Higher School and later Minister of Education. After graduating from the mathematics department, Makoto became an associate professor in the mathematics department of Tokyo Bunrika University a few years later. He had married Tae-ko, the elder sister of my wife. The Abe family had also evacuated to Yonezawa Village, but Makoto lived apart from them for a time, staying in Shigeno Village in Chiisagata District, Nagano Prefecture, where the mathematics department of Bunrika University had evacuated. He was a man of strong sense of responsibility and must have worked strenuously for the evacuation of the university; yet this was perhaps too much for his frail constitution. After returning in the summer to his family in Yonezawa Village, he took to his sickbed and died in October, at the age of thirty. It was a great loss. His thirty-eight papers were later published in a single volume by Iwanami Shoten.

I have no recollection at all of when or how the evacuated physics department returned to Tokyo after the war. I myself returned somewhat later, in the autumn. On the train journey back, one of my shoes was stolen. In pouring rain, with one foot bare and the other still shod, I made my way, consulting a map, to Risshisha in Chitose-Funabashi, where my father was staying. I lodged in one room of Risshisha and commuted to the University of Tokyo. As there was nowhere for my family to live in Tokyo, they remained in Yonezawa Village, and I resolved to return there at weekends.

Through the good offices of Professor Fujiwara Sakuhei, who was then Director of the Meteorological Observatory, my younger brother obtained a post at the Observatory. He too stayed at Risshisha and commuted to work. Later he was involved at the Meteorological Research Institute in the design of the meteorological satellite Himawari 1, and in April 1977 he became the first Director of the Meteorological Satellite Centre, taking part in the launch and observation operations of Himawari 1.

Before long, my father, my brother and I, together with Hanayo, moved into a barrack built on the burned-out site in Naka-Ochiai. Neither my brother nor I remember exactly when this was, but I believe we had already moved there by the summer of 1947.

Kazuhiko, my eldest son, who was in Yonezawa Village, contracted nephrosis, a serious and intractable illness, and in January 1947 he was admitted to the Red Cross Hospital in Kami-Suwa. My wife was five months pregnant at the time, yet from then on a six-mat hospital room became the home for her and Kazuhiko. At weekends I would go to Kami-Suwa and stay in that room, but I was plagued night after night by bedbugs (people nowadays may not know them; when crushed, they smell of peanuts). In those days, in hospital one had to bring charcoal and rice and cook one's own meals.

In May our eldest daughter was born. However, Kazuhiko's condition was such that it could suddenly worsen at any moment. We were greatly indebted to Dr Miyasaka, the head of paediatrics. At last the hospital fees were exhausted, and at the beginning of November we took him back to Yonezawa Village. On the 13th, his young life of three years and eight months came to an end. Our daughter Yasuko was six months old.

XIV The theory of curved spaces

Even after returning to Tokyo, the food shortage remained as severe as ever. For heating we had only a kotatsu, and power cuts were frequent. Only the houses requisitioned by the Occupation forces and inhabited by Americans were spared the outages. In Ginza there were restaurants reserved exclusively for the Occupation forces. I can still vividly recall one of them, with its glass façade: inside, beneath brilliantly shining chandeliers in what looked like a warm room, Americans were eating steaks with evident relish. It was impossible to imagine that Japan's economy would ever recover and prosper. I felt deeply dejected, thinking that Japan had sunk permanently to the rank of a fourth-rate nation.

And yet the students studied diligently. They were excellent. No matter how ingeniously I devised difficult examination problems, there were always several students who obtained full marks. They were utterly different from present-day university students, for whom one must rack one's brains to set easy questions for the year-end examinations. For some reason, there was a single lemon in the drawer of my desk in the physics department. At that time, lemons were precious commodities. It had grown blue mould, but its fragrance was unchanged. Sipping tea flavoured with slices of lemon cut as thinly as possible, I would continue the seminar until after eight o'clock in the evening – of course without any supper. It is a wonder to me how we all had such energy when we scarcely had enough to eat.

I have forgotten what we studied in that seminar, but I do remember that we worked through Heisenberg's paper on the theory of the S-matrix. Heisenberg had proposed the S-matrix theory on the philosophical principle that a physical theory should be constructed using only directly observable quantities. I only learned recently, to my astonishment, that this paper had been brought from Germany during the war by submarine, transported by the Japanese Navy.

That seminar led me to take an interest in the eigenvalue problem for second-order ordinary differential equations, and I discovered a general formula for eigenfunction expansions. By means of this formula, it became possible to prove at a stroke various expansion formulae in terms of special functions which in mathematical physics had previously been established separately. Moreover, when applied to the Schrödinger equation, it yielded a connection with Heisenberg's S-matrix. I therefore wrote up the results in an English paper entitled "The eigenvalue problem for ordinary differential equations of the second order and Heisenberg's theory of S-matrices", and in August 1948 I asked Professor Hideki Yukawa – who had been invited to the Institute for Advanced Study in Princeton – to hand it to Professor Weyl on my behalf.

Professor Weyl then wrote to inform me that Titchmarsh had already obtained the same formula by an entirely different method. It was a little disappointing, but with Professor Weyl's kind assistance the paper was published the following year, in 1949, in the *American Journal of Mathematics*.

To go back somewhat: from around the time I graduated from the physics department, I had been attracted by Weyl's theory of Riemann surfaces. I read his celebrated book *Riemann Surfaces* with great care and vaguely entertained the hope of extending it to higher-dimensional spaces. The section of *Riemann Surfaces* dealing with real harmonic functions is, in fact, the theory of harmonic vector fields in two real dimensions. I first attempted to extend this to higher dimensions and found that, by using de Rham's theorem, Hadamard's fundamental solution of partial differential equations, and Weyl's method of orthogonal projection, the extension could be achieved without difficulty. I therefore decided to postpone writing a detailed paper and instead summarised the results in an article entitled "Harmonic fields in riemannian manifolds", which Professor Teiji Takagi kindly arranged to have published in 1944 in the *Proceedings of the Japan Academy* (European-language edition).

For readers without a background in mathematics, a riemannian manifold may be thought of as a higher-dimensional version of the curved space that appears nowadays in popular explanations

of black holes; and a harmonic tensor field may be regarded as something like an electromagnetic field spread throughout such a space.

[レフシェッツ教授]

I continued writing the detailed paper in Suwa, where we had evacuated, and completed the final page in the hospital room of the Red Cross Hospital in Kami-Suwa, where my eldest son was hospitalised, tormented by bedbugs. This long paper remained as it was for some time, but in the following year, 1948, Shizuo Kakutani said that he had an acquaintance in the Occupation forces and would ask him to see that it was sent to the *Annals of Mathematics*. I could not help feeling some anxiety as to whether entrusting it to someone in the Occupation forces – who seemed to have little connection with mathematics – would ensure that it reached its destination safely. However, since there was no prospect of publishing it in Japan, I decided to accept his kind offer. Then, in October, I received notification from Professor Lefschetz, the editor-in-chief of the *Annals*, that the paper had been accepted for publication. It appeared in the 1949 volume of the *Annals of Mathematics*.

Even so, I still do not quite understand what possessed me to devote myself so earnestly to writing a paper for which there seemed to be no prospect of publication.

XV Princeton

I had heard that members of the Institute for Advanced Study in Princeton had no obligations whatsoever and were free simply to pursue whatever research they pleased. The thought that I might go from food-starved Japan to the food-abundant Institute in Princeton, America, and spend my time there was certainly an attractive one. Yet, being by nature indolent, it never even occurred to me to submit an application of my own accord.

In the spring of 1948, Professor Masao Sugawara told me that he had asked Professor Teiji Takagi to write a letter of recommendation for me to Professor Weyl at the Institute for Advanced Study. About half a year later, I received a letter from Professor Weyl inviting me to come to the Institute for one year from September 1949. I immediately went with Professor Sugawara to Professor Takagi's home to express my gratitude. Professor Takagi, however, said with perfect composure, "Heh heh... in fact, I've been rather idle and haven't written anything yet..." I remember being impressed, thinking that this must be how a great master conducts himself – so serenely unperturbed.

Professor Weyl's invitation must have stemmed from his interest in my paper on harmonic tensor fields. Had I not written that paper, for which there seemed no prospect of publication – or had Mr Kakutani not asked someone in the Occupation forces to forward it to the *Annals* – I should probably have remained in Japan and lived there contentedly, never going to America at all. It seems that a person's destiny can change completely through the smallest of chances.

The procedures for travelling to America were troublesome. At that time Japan was under American occupation, and it was not the Japanese government but General MacArthur's Headquarters that issued passports. Naturally, there were no such things as travel agencies, so I had to go in person several times to the American Consulate in Yokohama. For the visa I underwent a medical examination at St Luke's Hospital in Tsukiji. The idea was that no visa would be granted to an invalid.

The examination was fairly strict. It included a chest X-ray and even a test for roundworms, and I was made to take medicine to expel them. It was a powerful drug. Though it is an unpleasant story, on my way home a roundworm, in its distress, crawled out through my anus and slid down my leg inside my trousers before dropping to the ground. I felt something cool and clammy running down my leg, and it turned out to be a gigantic roundworm nearly thirty centimetres long.

On 9 August 1949 I sailed from Yokohama for America on the passenger ship *President Wilson*. I was travelling together with Professor Shin'ichirō Tomonaga, who had been invited by Oppenheimer to the Institute for Advanced Study. We were third-class passengers, and our cabin was a six-berth room with bunk beds.

[朝永振一郎さん(右)と著者(シカゴの空港にて)]

In Hawaii, the crew went on strike, so what had been scheduled as a one-day stopover was extended to three. A young physicist from the University of Hawaii, Mr Shimamoto, who had come to call on Professor Tomonaga, showed us generous hospitality. For three days he drove us about the island, treated us to Japanese and Chinese meals, and entertained us splendidly. According to him, there were no thieves in Hawaii, so there was no need to lock one's house when going out. I found this most impressive.

At the University of Hawaii, when we entered the office of a certain associate professor of physics, we saw copies of the specialist journal *Physical Review* on elementary particle theory tossed carelessly into a cardboard box. Mr Shimamoto said to Professor Tomonaga, "This professor never reads *Physical Review*, so if there's anything you want, please take it." Professor Tomonaga replied, "Let's see, then," and rummaged through the box, helping himself to three or four issues. It was all very easy-going.

It took two weeks to reach San Francisco. From there we flew to Chicago, where at the University of Chicago I had the opportunity to meet the great mathematician André Weil. Thanks to Professor Tomonaga's kindness, I was also introduced to the renowned physicist Enrico Fermi, who

treated us to luncheon.

From Chicago we travelled by train to New York. Princeton lies about an hour by car from New York. After staying in New York for about a week, we arrived in Princeton on 9 September – exactly one month after leaving Japan – driven there by a car sent from the Institute for Advanced Study. I was at once received by Professor Hermann Weyl and by the Director, Dr J. Robert Oppenheimer.

Professor Weyl was perhaps the last great mathematician of truly grand scale in this century. His range of research extended not only over mathematics but also into physics and philosophy. When Albert Einstein published general relativity, Weyl promptly wrote *Space-Time-Matter* and attempted a unified field theory; when quantum mechanics emerged, he produced *Group Theory and Quantum Mechanics*. In all, he left behind more than 160 papers and sixteen books – an immense body of work.

[ワイル教授(左)とエックマン教授(右)とともに(後年、チューリヒのワイル教授宅にて)]

When I first met Professor Hermann Weyl, he was a tall, round-faced, well-built gentleman, always smiling, with the air of a genial and kindly uncle. It was quite unexpected. He seemed astonished at how poor my English was; gazing intently at my face, he said, “When your English has improved in the second term, let us have a seminar.”

At that time the professors of mathematics at the Institute for Advanced Study were Weyl, Oswald Veblen, Marston Morse, Carl Ludwig Siegel, and John von Neumann. The permanent members were Kurt Gödel, James Waddell Alexander II, Deane Montgomery, and Atle Selberg, and in addition there were about forty short-term members on one-year contracts, such as myself. Just as I had heard, short-term members had absolutely no obligations; it made no difference at all whether one went to the Institute or not. The only requirement was that one reside in Princeton.

At that time Princeton was a small university town of about ten thousand inhabitants. On one side of the main street stood Princeton University, and on the other were restaurants, general stores and the like. The lodging that the Institute arranged for Professor Tomonaga and me was a small three-storey wooden house on the outskirts of the town. The Institute itself stood in woodland about two kilometres from the town. The building was U-shaped: the central main building had four storeys, and the two wings had two storeys each. The office assigned to me was on the second floor of one of the wings. It had a fine outlook and was most comfortable. The climate there was marked by sharp changes between heat and cold, yet even on the coldest days the room temperature was automatically maintained at twenty-two degrees Celsius. It was a splendid environment, unimaginable in Japan at that time.

XVI Institute for Advanced Study

Every morning at about ten o'clock I went to the Institute for Advanced Study. There was a bus from the town to the Institute, but even on foot it took only about twenty-five minutes. I would read books or write papers in my office, and at twelve o'clock I went up to the dining room on the fourth floor. Professor Hermann Weyl usually came to luncheon as well and dined together with us younger members.

My inability to understand English was a serious handicap. The Professor would apparently make various jokes, and everyone would laugh with evident amusement, while I sat there blankly. For those who had come from Europe, it seems that writing English was far more difficult than speaking it; they found it incomprehensible that someone could write papers in English yet be unable to speak it.

When I was asked, "Did you really write the English of your paper yourself?" I was at a loss for words. Professor Weyl seemed endlessly entertained by my inability to speak English. "Next year you shall speak in the seminar. Ha ha...", he would say, in high good humour.

[プリンストン高級研究所]

Yet even without understanding English, I managed well enough. All practical matters – banking, typing letters, and so forth – were taken care of by the secretary, Miss Eglehart. She had been born in Karuizawa, Japan, and had once taught music at Kōran Girls' School, where my wife had studied and from whom she had received lessons. This astonished me. I could not help reflecting that the world is, after all, a surprisingly small place.

The Institute for Advanced Study had its summer vacation from early May until late September, and from the end of September lectures and seminars gradually began. I attended Professor Carl Ludwig Siegel's lectures on the three-body problem. They were held three times a week for one hour each. He spoke in slow, deliberate English and seemed to carry even the most complicated formulae entirely in his head, lecturing without notes. Later, when I was at Johns Hopkins University, Professor Siegel came to give a colloquium talk, and afterwards we all went out for Chinese food. On that occasion he remarked casually, "I often begin studying mathematics at about nine in the morning, become completely absorbed, and forget to eat until midnight. When I then take a whole day's meals at once at midnight, my stomach does not seem to agree with it." I thought to myself that this was beyond imitation; it was far beyond the capacity of an ordinary person.

At the beginning of October I received word from Professor Donald C. Spencer of Princeton University that he wished to see me. When I went to meet him, he said he would like to hold a seminar on my paper on harmonic tensor fields. I declined, saying that I could not speak English. "But you have just said 'I cannot speak English' in English," he replied, and so it was decided that the seminar would go ahead. The first meeting took place in mid-October. When I heard the word "seminar", I had imagined that someone else would read my paper, but in fact I was expected to explain it myself – in other words, it was no different from giving a lecture. I could not possibly have imagined at the time that this seminar would mark the beginning of more than a decade of collaborative research with Spencer.

In November came the news that Professor Hideki Yukawa had been awarded the Nobel Prize. Professor Shin'ichirō Tomonaga and I went together to Professor Yukawa's home in New York to offer our congratulations. On our way back we were intercepted by a reporter from the *Mainichi Shimbun*, who asked the three of us – Professors Yukawa and Tomonaga and myself – to take part in a round-table discussion. Professor Tomonaga flatly refused, and the matter ended there. He thoroughly disliked appearing in newspapers.

The following evening, at a restaurant in Princeton, Professor Tomonaga and I drank a toast to celebrate Professor Yukawa's Nobel Prize. As the drink took effect, he grew talkative. He told me that when he had studied in Germany before the war, he often went to the cinema to pass the time, but began seeing the same newsreel twice and realised he was going too often; that he had once been

invited to dinner by Werner Heisenberg but declined because it seemed troublesome, and must have been thought an odd fellow; and many other such stories. Finally he said, "If I should ever receive the Nobel Prize, I'll give you half of it."

In mid-November there was the customary cocktail party at the home of the Director, J. Robert Oppenheimer, attended by almost all the members of the Institute for Advanced Study. At about seven o'clock, thinking it time to leave, I looked for Professor Tomonaga and found him in a corner industriously eating the hors d'oeuvres. "It's a nuisance to go back to the boarding house and then come out again for supper, so I'm eating here," he explained. On our way out, a young mathematician named Bateman came up beside us and said something. I assumed he was offering to drive us back to town, so I invited Professor Tomonaga to get into the car. Instead, we were taken to Bateman's house, where a second gathering began and dinner was served. I was taken aback. I had failed to understand the simple English words, "Won't you come to my house for dinner?"

Almost every evening we dined together at a restaurant in the town. By about the third month after arriving in Princeton, I had grown thoroughly weary of the American food that had seemed so enviable in food-starved Japan, and found myself craving Japanese cooking for no particular reason. At weekends we often went to New York and ate Japanese meals at somewhat dubious-looking restaurants.

We spent the New Year of 1950 in New York. On New Year's Day Professor Shin'ichirō Tomonaga and I called at the home of Professor Hideki Yukawa, where we were treated to *ozōni*, and shown his Nobel gold medal and certificate. On the evening of the following day, Professor Tomonaga and I were entertained at the home of a Japanese gentleman (whose name, I regret to say, I have forgotten) to a New Year's feast beginning with spiny lobster. It was a sumptuous Japanese meal such as could never have been obtained in Japan at that time, and I could not help thinking that New York was indeed the foremost city in the world.

However, from overeating and overdrinking Professor Tomonaga upset his stomach and caught a cold, and was quite dejected. He had all his teeth extracted and was fitted with a complete set of dentures, which made him look remarkably younger. Curiously enough, once he had American-made teeth, his Japanese seemed to deteriorate while his English improved. The price of the dentures, however, was no less than 250 dollars (90000 yen). At that time my monthly salary at the University of Tokyo had been just under 9000 yen, so 90000 yen was a considerable sum indeed.

Since falling ill at the New Year, Professor Shin'ichirō Tomonaga became somewhat faint-hearted and spoke constantly of returning to Japan out of homesickness. "I want to take off my shoes and go barefoot," he would say. "I want to chatter away in Japanese to my heart's content." "I've lost my *jintsūriki*." By this last expression he meant that no fresh and clever ideas were coming to him at all. When I remarked that it would be a pity to return home without first astonishing J. Robert Oppenheimer, he replied, "Unless I eat rice, no good ideas will occur to me."

At this stage Professor Tomonaga would still concede one merit of America: "At least the lavatories don't smell." Yet as summer approached, his comments escalated. "Even in summer there are no temple fairs," he complained. "The windows all have screens, so no mosquitoes come in. If I don't hear a mosquito buzzing, I don't feel that it's summer." And at last he declared, "A lavatory ought to smell after all!"

XVII Summer vacation

When I first met Professor Hermann Weyl, he had said to me, “When your English improves, let us hold a seminar.” That seminar began on 2 February. It was held every Friday from 9:40 to 11 am, and continued until April. In the first few sessions Professor Weyl spoke about historical matters; then Georges de Rham gave seven or eight talks on the theory of harmonic differential forms on riemannian manifolds, based on the concept of currents. Harmonic differential forms are another name for harmonic tensor fields. Afterwards I spoke several times on their applications to complex manifolds. At the final session de Rham spoke for an hour, I followed for an hour and a half, and then we all went into town for lunch. Professor Carl Ludwig Siegel raised a glass of wine and proposed a toast to our seminar.

On a Friday in mid-February, after the seminar, there was an unusual lecture by Albert Einstein. It was thought that, if word became widely known, a great crowd would descend, so the noticeboard merely read, “Lecture at eleven o’clock,” without giving either the title or the lecturer’s name. During the seminar the message was passed around in whispers: “There will be a lecture by Einstein at eleven – but it’s a secret.”

[ド・ラーム (左) と著者]

Einstein appeared without a jacket, wearing a high-collared coat, and began writing formulae on the blackboard while muttering something under his breath. At first I could not quite follow him, but listening carefully I realised that he was reading the letters in his formulae – a, b, c, ... – in the German manner: ah, beh, tseh... The substance of the lecture was that, if one allowed a non-symmetric metric tensor in general relativity, it was possible to construct a unified field theory including the electromagnetic field. At the time, such unified field theories based on differential geometry were considered old-fashioned, and young physicists paid them little attention; yet recently they seem to be studied vigorously once again. The fashions of scholarship are indeed a curious thing. When the seminar ended, the Institute for Advanced Study was already on its summer vacation, and Professors Weyl and Siegel departed for Europe.

On his way back to Japan, Professor Shin’ichirō Tomonaga was to meet Professor Ryōkichi Sagane in the university town of Ames, Iowa, and travel across the continent by car as far as San Francisco. I decided to accompany them. We left Ames on the morning of 27 June and, visiting places such as the Grand Canyon on the way, arrived in San Francisco on 9 July. Neither Professor Tomonaga nor Professor Sagane was particularly enthusiastic about sightseeing. Even at the Grand Canyon, when we came to a spot that seemed to offer a fine view, they would stop the car and tell me to go and take a look. I would go off on reconnaissance and report, “It’s a splendid view.” Only then would the two professors leisurely alight from the car and go to see it themselves.

In San Francisco I met Kenkichi Iwasawa, who had come from Japan to stay at the Institute for Advanced Study from September, and we travelled together to Chicago. We promptly called on Professor André Weil, and through his kindness Iwasawa and I were given offices in the Department of Mathematics at the University of Chicago, where we were to study until late August. We stayed at the International House, a student residence.

[ヴェイユ教授]

Professor Weil, whose family had gone to France, was also staying there alone, so we saw him many times each day. We usually had luncheon together, and on every occasion he would set us problems, which left us at our wits’ end. The brilliance of Professor Weil’s mind was astonishing: almost anything I tried to think through turned out to be something he had already considered before, and I could do nothing. I studied perhaps three times as hard as I had in Princeton. Iwasawa too was bombarded with questions, and said that never in his life had he studied so intensely.

Professor André Weil was fond of walking, and Iwasawa and I were often invited to accompany him. He walked extraordinarily fast and had the peculiar knack of darting across highways, weaving between cars that were hurtling past. I still recall him standing on the far side, grinning as he watched

us hesitating and unable to cross. It seemed that he was not satisfied unless he had walked several kilometres; moreover, he talked about mathematics incessantly as he walked, so these strolls were by no means easy.

At the International Congress of Mathematicians held at Harvard University from late August to early September, about 1700 mathematicians were in attendance. From Japan came Professors Suetsuna, Iyanaga, and Kōsaku Yosida. The Fields Medals at this congress were awarded to Laurent Schwartz of France and Atle Selberg of the Institute for Advanced Study in Princeton.

After the International Congress of Mathematicians had concluded, a conference on the theory of functions of several variables was held on 7 September, followed on the 8th by a conference on algebraic geometry. I was scheduled to speak at the meeting on the 7th, but Professor Oscar Zariski, the grand old man of algebraic geometry, asked me to give the same talk at the meeting on the 8th as well, and in the end I spoke at both. The algebraic geometry conference included many Italians, and the conversation would quickly turn into French or Italian, which left me quite at a loss.

Professor Wei-Liang Chow of Johns Hopkins University invited me to come from September for a year as a visiting associate professor. I felt rather apologetic, since at Professor Weil's request it had already been decided, fully six months earlier, that I should remain at the Institute for Advanced Study for another year. However, Professor Chow was so earnest in his invitation that, after consulting Professor Weil by letter in Switzerland, I finally decided to accept.

XVIII Reunion with my family

At the beginning of October 1950 I moved to Baltimore, where Johns Hopkins University is located. The university campus was compact, but the city of Baltimore was a large metropolis. I was somewhat anxious about whether I could lecture properly in English to students, yet once I stood at the lectern it proved to be nothing in particular. The only difficulty was that I often could not understand the questions.

In mid-June 1951 I returned to the Institute for Advanced Study in Princeton. At the end of the month my family arrived from Japan: my wife and our two daughters. The younger, who had just turned one, had been born after I came to Princeton.

On the wide lawn beside the Institute stood several rows of six terraced houses. These served as accommodation for short-term members with families. Our family occupied one of them. I was told that the buildings had been transported from disused miners' barracks, and people called them "monkey houses". On the ground floor there was a living room and kitchen; upstairs were two bedrooms; and in the centre of the ground floor stood a large oil stove. Having escaped from life in a makeshift shack amid the burnt ruins of Tokyo, my family rejoiced as though they had come to paradise. My wife was thrilled to be able to buy joints of beef. The lawns around the houses were surrounded by vast woods, where wild strawberries grew and deer would occasionally emerge; the children were delighted.

["マンキー・ハウス"の前で]

We promptly bought a second-hand piano. The price was a mere sixty dollars! It was half a tone flat, and we were told that tuning it would be impossible. My wife ordered a violin for ten dollars by mail order from Sears. The meteorologist Phillips, who lived next door, played the horn; the mathematician Lepson, in another of the houses, played the bassoon; and Alexander played the flute. It was decided that we should try some chamber music together, and we gathered at our house several times for that purpose.

I no longer remember clearly what we played, but I do remember the difficulty caused by the piano's being a semitone flat. In the trio by Johannes Brahms for piano, violin and horn, the horn was played a semitone lower. In the Horn Sonata by Ludwig van Beethoven, I transposed my part up by a semitone. On one occasion we played the Concerto in D minor for two violins by Johann Sebastian Bach: my wife took the first violin, and Phillips played the second violin part on the horn. Phillips's brilliant horn playing was of a very high standard. We ourselves enjoyed it immensely, though I imagine that the mathematician Kadison, who lived next door on the right, must have found it rather trying.

Beside the back lawn of the Institute stood a separate building housing the first computer built under John von Neumann. Phillips was conducting research there on weather forecasting by computer. On one occasion he took me to see the laboratory. The computer – constructed, as I recall, with 14000 vacuum tubes – occupied half of a large room. In those days it seems to have been an immense task to build such a machine and make it function properly, yet its performance did not match that of a commercially available pocket calculator today. It was about thirty-five years ago that I paid that visit. One cannot but be astonished at the rapid progress of computers, and at the same time pay tribute to von Neumann's foresight.

[ノイマンの初代コンピューター]

At that time the Institute for Advanced Study was divided into the School of Mathematics and the School of History. I had no dealings with the professors in the School of History, but on one occasion I was invited to a party given by one of them. The guest of honour was Mrs Elizabeth Vining, who had stayed in Japan after the war as tutor to the Crown Prince, and it seems that we were included because we were Japanese. Those present were all professors in the humanities, and before long the conversation turned to *The Tale of Genji*. Everyone had read and remembered the entire work, whether in English or German translation, and they chatted merrily: such-and-such

an episode occurs in “Kiritsubo”, another in “Yūgao”, and so forth. It appeared that *The Tale of Genji* was common knowledge among professors in the humanities; my wife and I were the only ones who had not read it.

[ある日のアインシュタイン]

After spending a year at the Institute for Advanced Study, I moved in September 1952 to Princeton University through Spencer’s good offices. The chairman of the Department of Mathematics there was the famous Professor Solomon Lefschetz. When my wife first met Professor Lefschetz at a party, he looked the two of us up and down and said to her, “You are taller than your husband.” Such was his greeting on our first meeting. At the Institute a young mathematician, Friedrich Hirzebruch, had arrived from Germany.

Around this time Mr Yoshishige Abe, who was then serving as President of Gakushūin in Japan, came to Princeton. He said that he wished to meet Albert Einstein and speak with him about the problem of peace. I thought it quite impossible, yet when I timidly made enquiries through the secretary at the Institute for Advanced Study, the meeting was arranged with surprising ease. Thus I went with Mr Abe and a Japanese interpreter to visit Einstein, and listened to their conversation. I no longer recall what Einstein said in detail; I only remember vividly his remark that he was extremely pessimistic about the prospects for world peace.

I learned afterwards that Albert Einstein was an affable man. On one occasion, when my wife and I were standing with our daughters in front of the left-hand entrance of the main building of the Institute, Einstein happened to pass by. He deliberately came over and shook hands with the children.

It seems that he would readily agree if anyone asked, “May I take your photograph?” The photograph on the previous page was taken by the physicist Yoichiro Nambu. When Minoru Kobayashi tried to photograph him, he was so nervous that he mishandled the camera, and the picture was a failure.

Under Spencer’s guidance a seminar on sheaves began. It was a young student with the unusual hobby of buying dilapidated second-hand cars and repairing them himself who read the notes of the Cartan Seminar in this seminar; from time to time he arrived late because his repairs had not been completed in time. My first impression was that a sheaf was something abstract and rather strange, lacking any tangible substance. It was not until the spring of 1953 that I began to understand how useful sheaves could be.

I lunched with Spencer almost every day, and we would talk about mathematics. One day at luncheon we realised that, by using sheaves, Severi’s conjecture – that the two kinds of arithmetic genus, P_a and p_a , coincide – could be proved with remarkable ease. In a lecture delivered in 1949 on the algebraic geometry of the Italian school, Francesco Severi had emphasised the difficulty of resolving this conjecture by comparing it to a star shining far in the distance. We promptly published a joint paper entitled “Arithmetic genera of algebraic varieties.”

Thus it became clear that sheaves were extraordinarily effective in algebraic geometry and in the theory of complex manifolds, and Spencer and I applied them to a variety of problems, producing a number of joint papers.

In *Riemann Surfaces* by Hermann Weyl, the most important theorem is surely the Riemann–Roch theorem. At that time, the central problem in the theory of complex manifolds was to extend this theorem to higher dimensions. Late in the autumn of that year, Friedrich Hirzebruch solved this central problem. With that, it felt as though the general theory of complex manifolds had reached a natural stopping point.

XIX Fields Medal

Since 1897 the International Congress of Mathematicians has been held every four years. The first took place in Zurich, Switzerland, and the second in Paris in 1900. At that congress David Hilbert delivered his famous lecture entitled “Mathematical Problems”, in which he set out twenty-three representative problems and thereby indicated the aims of twentieth-century mathematics.

At the International Congress of Mathematicians held in Zurich in 1932, the Fields Medal was established as an international mathematical award. The Fields Medal is presented at each congress to young mathematicians of outstanding achievement, as a form of encouragement. At the first award ceremony, in Oslo in 1936, the prize went to Lars Ahlfors and Jesse Douglas; at the second, as mentioned earlier, it was awarded to Laurent Schwartz and Atle Selberg.

The 1954 International Congress of Mathematicians was scheduled to be held in Amsterdam in the Netherlands at the beginning of September. Being indolent, I had no intention of attending. Then, through Spencer, I received a message from Professor Hermann Weyl informing me that I had been selected as one of the recipients of that year’s Fields Medal. Professor Weyl was chairman of the Fields Medal Committee on that occasion.

For some reason, the names of the recipients were kept strictly secret until the day of the award ceremony. I do not recall when I learned that the other recipient was Jean-Pierre Serre.

In order to attend the International Congress of Mathematicians, which began on 2 September, my wife and I left New York in mid-August. We decided to take the opportunity to see a little of Europe. We first went to Italy and visited Rome, Naples, Capri and Pompeii. At that time there were remarkably few motor cars even in Rome, and motorbikes sped about through the spacious streets. The breakfast we ate on the terrace of a hotel perched on the cliffs of Capri, gazing down at the deep blue sea far below, was magnificent.

Next we went to Switzerland. In Lausanne we met Georges de Rham and walked together along the shores of Lake Léman, and visited the Château de Chillon. He remarked that in most years the Alps were clearly visible, but that this year there were too many clouds – surely, he said, it must be due to atomic bomb tests. De Rham was also a master mountaineer. When we visited his house he showed us a mountaineering journal, and he himself was one of its editors. In the evening he treated us to dinner at a restaurant. In Zurich we were invited to luncheon at the home of Professor Hermann Weyl, where I met Mrs Weyl and their daughter. She told us that she was studying music at Yale University in America.

On the day we arrived in Amsterdam, we lost our way en route to the congress venue. My wife asked at a nearby greengrocer’s, and a young assistant explained the way in English. A guidebook we had bought in Princeton declared that in Amsterdam even dogs understand English, so there is no need to worry – and indeed it proved true. At the venue we met Professor Bartel Leendert van der Waerden, the secretary of the congress, who explained various arrangements to us.

The Fields Medal award ceremony formed part of the opening ceremony of the International Congress of Mathematicians. About 1500 mathematicians were gathered in the hall. Among them were several mathematicians from Japan, including Professors Iyanaga and Kōsaku Yosida. I entered the hall and sat in the front row as instructed in the letter I had received. Apparently I was taken for some odd little Oriental who had mistakenly sat in the wrong place, for an usher came up to me and whispered that this was a reserved seat and that I should move to the back. When I showed him the letter of instruction, he apologetically withdrew.

[フィールズ賞会場、最前列に座る著者(左から3人目).]

[ワイル教授からメダルを受ける著者]

[授賞式でのセール、著者、ワイル教授]

[著者とセールの業績を説明するワイル教授]

After the opening address by the chairman, Professor Jan Arnoldus Schouten, there was a piano performance by Mrs Fania Chapiro. The programme consisted of a Chopin Impromptu, a Noc-

turne and a Scherzo. Next came the award ceremony. From Professor Hermann Weyl, chairman of the Fields Medal Committee on that occasion, Jean-Pierre Serre and I each received a gold medal and a prize of 1500 dollars. Professor Weyl then spoke for more than an hour, giving a detailed account of the achievements of the prize-winners. After that there was another piano performance, followed by a brief speech from Enrico Bompiani, Secretary of the International Mathematical Union. With that, the opening ceremony came to an end.

XX International Congress of Mathematicians

After the opening ceremony, at three o'clock in the afternoon, there was a lecture entitled "Unsolved problems in mathematics" by John von Neumann, the pioneer of the computer. I went to hear it in the expectation that it would rival the celebrated lecture "Mathematical problems" by David Hilbert. However, he merely mentioned several problems concerning Hilbert space. Everyone was disappointed. Hilbert space is but one small branch of mathematics. It was difficult to understand why a figure of von Neumann's stature should have given so uninspiring a lecture.

I had been anxious lest, as a recipient of the Fields Prize, I might be required to give a special award lecture, but this did not occur. Since the names of the prize-winners were kept secret, it was probably impossible to incorporate such lectures into the programme. According to the schedule, I delivered my talk on the third day, from 12:30 to 1 pm, under the title "Some results in the transcendental theory of algebraic varieties". Jean-Pierre Serre spoke on the second day, from 4:40 to 5:10 pm, on "Cohomology and algebraic geometry".

[セヴェリ教授とその姪]

The grand old man of Italian algebraic geometry, Francesco Severi, attended with a lady whom he introduced as his niece. Some years earlier Severi had come to Japan from Italy as a cultural envoy and delivered a series of lectures in a tiered classroom at the University of Tokyo. At that time I was a student in the Department of Mathematics. I attended the lectures, yet remember nothing of their content. What I do remember is the splendid shape of the bald head of Professor Nakagawa, who was seated in front of me. Nevertheless, because I had attended those lectures, Severi would say, "Kodaira is my pupil." This came as a great surprise to me.

[フィールズ賞受賞後, 女王 (前列右から2人目) を囲んで (前列右端が著者)]

On 8 September, about a dozen eminent mathematicians – among them Hermann Weyl, Francesco Severi, W. V. D. Hodge and John von Neumann – together with us prize-winners were invited to tea by Juliana. The venue was the garden of a royal residence on the outskirts of Amsterdam, where chairs and tables had been set out in one corner of a broad expanse covered with neat gravel. Jean-Pierre Serre, who had absent-mindedly begun smoking, said, "What shall I do with this cigarette end?" I suggested, "Bury it in the gravel." "Like a cat?" he replied with a laugh.

I had gone imagining the queen as portrayed by Audrey Hepburn in the film *Roman Holiday*, but the real queen was an ordinary middle-aged lady. She asked Serre, "What do you usually do?" When he answered, "I usually teach," she turned to me and asked, "And what do you do?" "I also teach," I replied, whereupon she looked rather bored. After tea we took a photograph with the queen in the centre.

There were several symposia on algebraic geometry. At one of them Beniamino Segre won applause by declaring, "The problems we are discussing here today are far more important than von Neumann's problems."

On the evening of the third day there was a concert at the Royal Concertgebouw Orchestra. I was greatly impressed by the excellence of the acoustics. The programme included Bach's Second Suite, Mozart's Piano Concerto in F major, and Debussy's *La mer*. In the foyer I met Professor Weyl, who remarked that the piano playing was perhaps a little too showy.

At the banquet on the final evening, the speeches went on and on until midnight. Pavel Alexandrov of the Soviet Union first spoke in Russian and then repeated the same speech in German, which struck me as curious. Since the Soviet Union was a fearsome country, I imagined that he might be reprimanded afterwards if he spoke only in German.

[左から弥永昌吉先生, 著者夫人, 著者.]

After the congress, I went to Paris together with Professor Iyanaga. He seemed delighted, as though returning to his native home. We were shown around Paris by his close friend Claude Chevalley. Chevalley treated us to luncheon at a restaurant. Wishing to try something unusual, I ordered calf's brains; they tasted rather like tofu mixed with oil, and I cannot say that I found

them especially delicious.

We stayed three nights in Paris and then went on to London. The hotel crockery was splendid, but the cooking was poor. The following day we took the train to Cambridge, where Professor W. V. D. Hodge showed us around University of Cambridge. Hearing such explanations as, “This is where Newton carried out such-and-such an experiment,” I was duly impressed by the venerable traditions of the university. At a club we met Mrs Hodge and their daughter, who was the very image of her father, and were treated to tea. Across from the club stood the house of Charles Darwin.

[ホッジ教授と(ケンブリッジ大学にて)]

Mrs Hodge said thoughtfully to my wife, “Aren’t mathematicians odd!” If so distinguished an English gentleman as Professor Hodge was considered odd, I began to wonder what that might imply about me.

We returned to Princeton in mid-September. I remember feeling somehow relieved when we landed at the airport in New York and boarded the bus for the city.

In the following year, on the night of Sunday, 17 April 1955, Albert Einstein died at Princeton Hospital. Had this occurred in Japan, a grand institute funeral would surely have been held; but nothing of the sort took place. At tea-time in the Institute, the only exchange was along the lines of, “I hear Einstein died last Sunday.” “Oh, did he?” – and that was all.

According to a recently published biography of Einstein – Abraham Pais, *Subtle is the Lord – The Science and the Life of Albert Einstein* (Oxford University Press, 1982; Japanese edition supervised by Kazuhiko Nishijima) – Einstein in fact died at 1:15 am on 18 April, the cause being the rupture of an aneurysm. His body was taken that afternoon to the crematorium, where twelve people who had been close to him gathered, and one of them recited a passage from a poem by Johann Wolfgang von Goethe. The body was then immediately cremated, and the ashes were scattered at a certain place; it is said that the location has not been made public.

XXI The psychology of discovery

In September of that year I became a professor at Princeton University. To say “professor”, however, did not mean a full professorship; I held the rank of professor but was in fact a research appointee. At the same time, I became a member of the Institute for Advanced Study. My appointment there was for five years. In the autumn term I gave three hours of lectures a week at the University. In the spring term I was at the Institute and had no teaching obligations. Even at the University the lectures were for advanced graduate students, and I was told that I might lecture on whatever I pleased. I always chose to lecture on whatever subject I was working on at the time.

At the University of Tokyo, I had been promoted in 1951 from Associate Professor in the Department of Physics to Professor in the Department of Mathematics; but in April of this year I submitted my resignation and stepped down.

Around this time I began my research on elliptic surfaces. It was genuinely enjoyable to apply the general theory of complex manifolds in order to investigate the structure of elliptic surfaces in detail. Classical elliptic function theory proved astonishingly effective, and the research advanced steadily without any difficulty. The feeling one has at such a time is, I think, well expressed in the episode of Unkei carving a guardian deity in *Ten Nights of Dreams* by Natsume Sōseki. To quote a portion:

Unkei was now carving out a bushy pair of eyebrows, about an inch high. No sooner would he bring the blade of his chisel back up than his hammer would come down at an angle to strike it again. A stubby chip fell as each blow rang out, and as I watched an enraged nose emerged from the hard wood, nostrils flared. Unkei showed no hesitation as he wielded the blade. He did not appear to be troubled by even the smallest of doubts.

“Amazing that he can just throw the chisel around like that and still get the eyebrows and noses to come out the way he wants,” I said, almost to myself, as if too impressed to keep my thoughts inside.

“Oh, it isn’t the chisel that makes those eyebrows and noses,” the young man said. “Those exact eyebrows and noses are buried in the wood, and he just uses the hammer and chisel to dig them out. It’s just like digging a rock out of the ground – there’s no way to get it wrong.”

[スペンサー教授, 1959年]

My theory of elliptic surfaces was not, in truth, something that I invented. Rather, it was already buried within the tree called mathematics, and I merely dug it out with the aid of paper and pencil – that was my genuine feeling.

I wrote about this sensation earlier in *Introduction to Mathematics* (数学のすすめ, Chikuma Shobō, 1965), and in the New Year’s edition of the *Nihon Keizai Shimbun* in 1986 I noticed that Kenichi Fukui expressed a similar impression. It would seem that the psychology of discovery does not depend upon one’s particular academic field.

From the autumn of 1956, I began joint research with Spencer on the deformation of complex structures. We first examined readily accessible examples of complex manifolds and inductively derived certain laws concerning their moduli numbers. Then, taking these laws as working hypotheses, we developed the theory of deformations while testing numerous concrete examples. The process was extraordinarily interesting. I met Spencer every day; we lunched together at a restaurant in town, then returned to the University to discuss deformation theory. Examining concrete examples corresponds, in physics, to conducting experiments. At the beginning, deformation theory was an experimental science.

Donald C. Spencer was an American, born in 1912 in Boulder, Colorado. His body, trained by mountaineering in the Rockies, was the picture of robustness; he was tall and weighed over one hundred kilograms. After graduating from Massachusetts Institute of Technology, he went to England and studied diophantine approximation under J. E. Littlewood at University of Cambridge, taking his doctorate there. Before coming to Princeton he had been working on function theory at Stanford University. After arriving in Princeton he turned to the theory of complex manifolds and rapidly became a leading authority in the field.

According to what Spencer told me, it was only after coming to Princeton that he first learned of the existence of harmonic tensor fields. Sensing that something important lay there, he set himself to study with the intensity of a student beginning anew. Spencer possessed this kind of foresight. It was also Spencer who first proposed holding a seminar on sheaf theory. He was a man overflowing with goodwill and enthusiasm, and that enthusiasm “infected” those around him, so that a naturally dedicated research group formed. I believe that the driving force behind the rapid development of the theory of complex manifolds in Princeton during the 1950s was Spencer’s enthusiasm.

XXII Immersed in music

In the summer of 1956 we bought a house with the help of a loan. It was a brick-built house with a floor area of about eighty tsubo and grounds of roughly two hundred tsubo. The price was only 18000 dollars.

My eldest daughter's piano teacher, Miss Maclean, had taught at a certain Christian music school, but had been dismissed for drinking too much beer – a delightful old lady who was already over eighty at the time. She possessed an exceptional musical sensibility and was on close terms with the famous pianist Robert Casadesus and his family; she had even taught Casadesus's daughter. Through her good offices I purchased, from a wealthy elderly gentleman, a Steinway middle grand piano for the modest sum of 800 dollars.

Miss Maclean often came to our house and played piano duets with me. Thanks to her, I learned how to simplify difficult passages appropriately when sight-reading. On one occasion I went to her studio and played *Scaramouche* for two pianos by Darius Milhaud. There was no possibility that I could sight-read *Scaramouche* properly on my own; yet when I played together with her, it somehow became possible (with suitable simplifications). She had a mysterious power to draw people along with her. My second daughter's violin teacher, Kovács, also visited us from time to time. We once made a tape recording of Kovács and Miss Maclean playing a violin sonata by Ludwig van Beethoven. While they were playing, it seemed that Miss Maclean was omitting quite a number of notes; yet when we later listened to the tape, it sounded remarkably splendid. She must have secured all the essential points – and even so, it was something of a mystery.

There was a town orchestra in Princeton called the Princeton Symphony Orchestra. Kovács was the concertmaster, and the conductor was a Hungarian named Harsanyi. The members included various residents of the town – Professor Hassler Whitney of the Institute for Advanced Study, the proprietor of a stationery shop, and others. My wife also occupied a seat at the very end of the second violins. The owner of the stationery shop had originally hoped to become a violinist, but as one cannot make a living from the violin, he had taken up the stationery business instead; in fact, he was a professional.

This symphony orchestra gave concerts several times a year, selling tickets to the public. On such occasions they would invite a few members of the New York Philharmonic and place them in key positions, so that the performance sounded most impressive. The programmes were skilfully arranged, selecting works that were not technically beyond the amateur members. They did not attempt major works such as symphonies by Johannes Brahms. To force their way through large-scale works would only result in boring the audience. In this respect their attitude seemed quite different from that of amateur orchestras in Japan. What has remained most vividly in my memory is the waltz from *Der Rosenkavalier* by Richard Strauss. The performance was magnificent. It sounded like a first-rate orchestra. No doubt this was due to the Hungarian blood of the conductor Harsanyi – the playing was truly schmalzig, richly and almost cloyingly expressive, and I admired it greatly.

The final few bars of *Der Rosenkavalier* contain a rapid passage which was beyond an amateur such as my wife. At rehearsal, Harsanyi is said to have instructed the amateur members seated towards the back of the second violins: "Please, just pretend to play here – do not actually produce any sound." Quite reasonable, of course; no member of the audience watches the rear of the violin section with particular attention, so this would suffice. However, after hearing this story from my wife, I – who was looking only at the back rows – could clearly tell that no sound was being produced.

At Princeton University there was a Department of Music which taught music theory and composition and published a journal of music theory called *Perspectives of New Music*. On one occasion I went to a concert of works composed by students of the department. The performers were a small chamber orchestra. It was a curious kind of contemporary music in which the silences lasted longer

than the sounds; I could make nothing of it. One of the members of the chamber orchestra was a Japanese violinist whom I knew. During the interval I asked him what he thought of it. "I don't understand it either!" he replied. Music that the performers themselves do not understand can hardly be comprehensible to those who listen. Professor Ralph H. Fox of mathematics had reached a professional level as a pianist. He was seated next to me, so I asked him, "What on earth is this music?" He answered, "It is influenced by Zen – one is meant to listen to the sound of silence."

Princeton was a small university town of a little over ten thousand inhabitants, yet it possessed a small theatre where concerts were frequently held. As soloists I heard Rudolf Serkin, Isaac Stern, Andrés Segovia, Myra Hess, and Mieczysław Horszowski; among string quartets, the Budapest String Quartet and the Juilliard String Quartet; and as orchestra, the Philadelphia Orchestra. As tickets were easy to obtain, I always listened from seats in the third or fourth row from the front.

When Stern played Bach's Chaconne, I could see that three hairs of his bow had snapped and were hanging loose during the performance; and when he had finished and was bowing, I heard him say softly, "Thank you, thank you." When the Philadelphia Orchestra performed *La valse* by Maurice Ravel, the sound was so loud from the third row that I could scarcely make sense of it.